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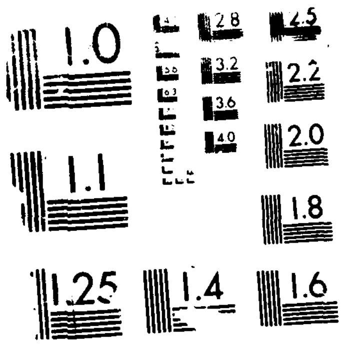

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Detection Performance of Normalizer for Multiple Signals Subject to Partially Correlated Fading With Chi-Squared Statistics

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Preface

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The false alarm and detection probabilities for a multi-pulse signal subject to partially correlated fading, in the presence of Gaussian noise of unknown level, are derived in closed form. The number K of signal pulses, as well as the number L of noise-only pulses used to estimate the noise background power level, are arbitrary. The power-fading is characterized by a chi-squared distribution with 2m degrees of freedom and a normalized set of covariance coefficients $\{\rho_{kj}\}$, all of which can be selected arbitrarily, in order to match an experimental realization or an actual measured situation. The performance capability of this processor depends additionally on the received signal-to-noise ratio.

This study is an extension of NUSC Technical Report 7707, to cover the case of a nonconstant threshold. Comparisons of this normalizer with the earlier results (for L = ∞) enable a quantitative evaluation of the losses incurred by lack of —) over

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Constant False Alarm Rate
Operating Characteristics

19. ABSTRACT (Cont'd.)

knowledge of the noise level. The important capability of constant false alarm rate is achieved by this normalizer.

Plots of the detection probability vs false alarm probability are furnished for a variety of typical choices of the various parameters; however, the multitude of parameters and cases precludes a comprehensive all-encompassing compilation of numerical results. Accordingly, a general program in BASIC is listed, whereby additional results of interest to a particular user can be easily obtained, once numerical values are assigned to all the parameters.

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LIST OF SYMBOLS

K	number of signal pulses added, figure 1
L	number of noise-only pulses, figure 1
m	fading parameter, table 1
ρ_{kj}	normalized covariance coefficient, table 1
ROC	Receiver Operating Characteristic
SNR	Signal-to-Noise Ratio
FFT	Fast Fourier Transform
$\overline{E_1}$	average received signal energy per pulse
N_o	single-sided noise spectral density level (watts/Hz)
K_e	equivalent number of samples, table 2
N	summary parameter, table 2
γ	sum of K signal squared-envelope samples
σ_n^2	noise power
a,b	auxiliary constants, (2)
R	scaled signal-to-noise ratio, (2)
Q_γ	exceedance distribution function of γ
Λ	scaled threshold, (3)
$E_n(x)$	exceedance function, (5)

LIST OF SYMBOLS (Cont'd)

$e_n(x)$	partial exponential, (6)
γ_o	sum of L noise-only squared-envelope samples
υ	normalizer ratio output, (9)
$\tilde{\upsilon}$	alternative normalizer ratio, (10)
u	threshold, (11)
P_υ	cumulative distribution function of υ
$f(\xi)$	characteristic function, (12)
p(u)	probability density function, (13)
$H_\ell(x)$	hypergeometric function, (25)
P_D	detection probability, (30)
P_F	false alarm probability, (31)
ρ	exponential correlation coefficient, (32)

DETECTION PERFORMANCE OF NORMALIZER FOR MULTIPLE SIGNALS SUBJECT TO PARTIALLY CORRELATED FADING WITH CHI-SQUARED STATISTICS

INTRODUCTION

In a recent study [1], the detection performance capability of a multiple-pulse system subject to correlated fading was quantitatively delineated. It was assumed there that the noise level was known, so that a threshold could be set for an arbitrarily specified false alarm probability. Then the detection probability was evaluated as a function of the threshold level, the received signal-to-noise ratio, the number K of signal pulses, and the fading statistics.

Here we will extend these earlier results to cover the case where, additionally, the noise level is unknown and must be estimated on the basis of a finite number L of noise-only samples. The same approximation technique that was presented in [1] is used to determine the detection probability of this normalizer system. The reader is referred to [1] for additional background, motivation, interpretations, and related references. For the sake of brevity, we will employ the same notation and presume that the reader has complete familiarity with the earlier material and development.

PROBLEM DEFINITION

We will couch the problem in a particular setting, one with obvious appeal and application; however, it should be obvious how to extend this setting to a more general one, particularly in light of the arbitrary fading covariance coefficients that are allowed in the analysis.

Suppose a sequence of K tone bursts at a common center frequency are transmitted, as depicted in figure 1. Each rectangular slot symbolizes

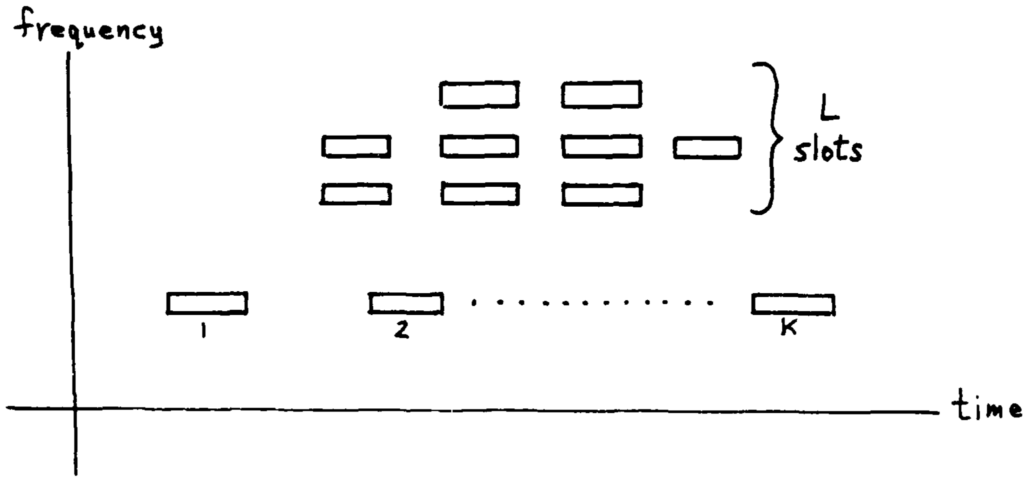

Figure 1. Time-Frequency Occupancy Diagram

a tone of duration T_1 seconds and approximate frequency bandwidth $1/T_1$ Hz. These bursts may be abutting in time or may be arbitrarily separated in time by several multiples of T_1. At the receiver, K narrowband filters of bandwidth $1/T_1$ Hz are sampled at the times of peak signal output (if present) and their squared envelopes are summed. Depending on the time separation between pulses, the signal strength may fade considerably; the

exact amount and frequency of the fading depends on the distribution of the fading and the covariance of the fading amplitude of adjacent (as well as separated) pulses.

It is presumed that, during a single tone of duration T_1 seconds, the fading is essentially constant, resulting in a constant amplitude scaling and phase shift applied to the pulse. The time separations between pulses in figure 1 are arbitrary, thereby allowing for an arbitrary degree of correlation between the fading factors applied to each pulse.

To establish a reference against which this sum of K matched filter outputs can be compared, for purposes of deciding on the presence or absence of signal, a group of L nonoverlapping noise-only slots, located arbitrarily in the time-frequency plane, are also energy-detected and summed. For very large L, this noise reference is very stable, and performance approaches that predicted by [1]. However, for moderate values of L and for small false alarm probabilities of interest, it is important to know how much degradation in performance is incurred by being forced to use this noisy reference.

An obvious implementation of the processing implied by figure 1 is to employ fast Fourier transforms. The L reference bins can then be an arbitrary collection of time and/or frequency bins. However, L cannot be so large that nonstationary and/or nonwhite noises cause their own kind of errors in noise power estimation. The tradeoff between these conflicting requirements will be assessed quantitatively in this investigation.

NORMALIZER PROBABILITIES

DEFINITIONS OF PARAMETERS

Very heavy reliance will be made here on the basis that was set up in [1]. Thus we have the following fundamental parameters of the detection procedure (the immediate references in tables 1 and 2 are to [1]):

K, number of potential-signal pulses added, (figure 1 and A-11);

m, signal fading parameter (power-scaling is chi-squared with 2m degrees of freedom), (13);

$\{\rho_{kj}\}$, normalized covariance coefficients of signal power-scalings $\{q_k\}$, (15);

$\frac{\overline{E_1}}{N_0}$, $\frac{\text{average received signal energy per pulse}}{\text{single-sided received noise spectral density level}}$, (9);

L, number of noise-only pulses added.

Table 1. Fundamental Parameters

In addition, there are two very useful auxiliary parameters that found frequent use in [1]:

K_e, $K^2 \Big/ \sum_{kj=1}^{K} \rho_{kj}$ = equivalent number of independent signal pulses, (10);

N, $m\,K_e$ = a summary parameter describing the distribution of the sum of power scalings, (A-24) and (A-29).

Table 2. Auxiliary Parameters

None of the parameters, m, K_e, N, need be integer. Also, N can be larger or smaller than K, the number of signal pulses.

PROBABILITIES FOR KNOWN NOISE LEVEL

The probability density function of the sum γ [1; (A-11)] of the K signal envelope-squared samples is given by [1; (B-4)]

$$p_\gamma(u) = \frac{\exp(-u/a)\ u^{K-1}}{a^{K-N}\ b^N\ \Gamma(K)}\ {}_1F_1\left(N;\ K;\ u\left(\frac{1}{a} - \frac{1}{b}\right)\right) \quad \text{for } u > 0\ , \tag{1}$$

where [1; (A-32),(B-3),(B-7)]

$$a = 2\sigma_n^2\ , \quad b = 2\sigma_n^2\ (1 + R)\ , \quad R = \frac{\overline{E}_1}{N_0}\ \frac{K}{N}\ . \tag{2}$$

The exceedance distribution function $Q_\gamma(u)$ of output sum γ is given by several alternative forms in [1; (B-9),(B-11),(B-13)]. For a fixed threshold (known noise level), the detection probability is

$$P_D = Q_\gamma(\Lambda, R, N, K) =$$

$$= 1 - \frac{1}{(1 + R)^N} \sum_{n=0}^{\infty} \frac{(N)_n}{n!} \left(\frac{R}{1 + R}\right)^n \left[1 - E_{K-1+n}(\Lambda)\right] , \Lambda = \frac{u}{2\sigma_n^2} , \tag{3}$$

and the false alarm probability is [1; (B-10)]

$$P_F = E_{K-1}(\Lambda) , \tag{4}$$

where we define the exceedance distribution function

$$E_n(x) = \exp(-x)\ e_n(x) , \tag{5}$$

and

$$e_n(x) = \sum_{j=0}^{n} x^j/j! \tag{6}$$

is the partial exponential [2; 6.5.11]. The results in (3) and (4) should be used for $L = \infty$, that is, for known noise level.

NORMALIZER RATIO

From this point on, L is presumed finite. Suppose a noise level estimate, γ_o, is obtained, based upon L independent measurements of *noise-only* bins. It is assumed that the average noise level in these L bins is the same as in the K potential-signal bins, but that this noise level is unknown. If we let

$$\gamma = \gamma(K, \bar{E}_1/N_0) \quad (7)$$

denote the sum of K signal bin outputs with average signal-to-noise ratio $\bar{E}_1/N_0$, then

$$\gamma_0 = \gamma(L, 0) \quad (8)$$

is the corresponding sum of L noise-only bins. Now define ratio

$$v = \frac{\gamma}{\gamma_0} = \frac{\gamma(K, \bar{E}_1/N_0)}{\gamma(L, 0)} \quad (9)$$

for sets of K and L pulses, respectively. The noise contributions to the total of K + L outputs are presumed independent of each other; however, the signal fading factors amongst the K signal outputs are correlated to an arbitrary degree. We are interested in the distribution of this normalizer ratio, v.

When signal is absent, the ratio v in (9) is independent of the absolute level of the received noise; therefore, we can expect the normalizer to achieve the important capability of constant false alarm rate. That means that a specified false alarm probability can be achieved without knowledge of the average noise power level.

The quantities γ and γ_0 are the sums of K and L squared-envelope samples, respectively, and are not the averages of these sampled quantities. In terms of the sample-average quantities, we could define a slightly different normalizer ratio

$$\tilde{v} \equiv \frac{\gamma/K}{\gamma_0/L} = \frac{L}{K}\, v \;. \qquad (10)$$

It then readily follows that the cumulative distribution function of random variable $\tilde{v}$ is

$$P_{\tilde{v}}(u) = \mathrm{Prob}(\tilde{v} < u) = \mathrm{Prob}\left(v < \frac{K}{L}\, u\right) = P_v\left(\frac{K}{L}\, u\right) , \qquad (11)$$

in terms of the cumulative distribution function of ratio v in (9). Thus, a simple scale factor change allows for consideration of the alternative ratio $\tilde{v}$.

When we plot the detection probability versus the false alarm probability, that is, eliminate the threshold, the same performance characteristics result for random variable v as for $\tilde{v}$. Accordingly, we will not use or refer to $\tilde{v}$ or $P_{\tilde{v}}(u)$ any further, but concentrate solely on normalizer ratio v, given by (9).

NORMALIZER DISTRIBUTIONS

The characteristic function of noise-only random variable γ_0 can be found directly from [1; (A-13)] by setting A to zero and replacing K by L:

$$f_{\gamma_0}(\xi) = (1 - i\xi a)^{-L} \;, \quad a = 2\sigma_n^2 \;. \qquad (12)$$

The corresponding probability density function of γ_0 is

$$p_{\gamma_0}(u) = \frac{u^{L-1} \exp(-u/a)}{\Gamma(L)\, a^L} \quad \text{for } u > 0 . \tag{13}$$

The exceedance distribution function is

$$Q_{\gamma_0}(u) = \text{Prob}(\gamma_0 > u) = E_{L-1}(u/a) \quad \text{for } u > 0 , \tag{14}$$

in terms of the functions defined in (5) and (6).

The cumulative distribution function of ratio υ in (9) is given by (since $\gamma_0 > 0$)

$$P_\upsilon(u) = \text{Prob}(\upsilon < u) = \text{Prob}\left(\frac{\gamma}{\gamma_0} < u\right) = \text{Prob}\left(\gamma < u\gamma_0\right) =$$

$$= \int_0^\infty dy\; p_\gamma(y) \int_{y/u}^\infty dx\; p_{\gamma_0}(x) = \int_0^\infty dy\; p_\gamma(y)\; Q_{\gamma_0}(y/u) =$$

$$= \int_0^\infty dy\; \frac{\exp(-y/a)\; y^{K-1}}{a^{K-N}\, b^N\, \Gamma(K)}\; {}_1F_1\!\left(N;\, K;\, y\left(\frac{1}{a} - \frac{1}{b}\right)\right) E_{L-1}\!\left(\frac{y}{ua}\right) \tag{15}$$

for threshold $u > 0$, where we used (1) and (14). We now expand E_{L-1} according to (5) and (6) and integrate term-by-term,to obtain [3; 7.621 4]

$$P_\upsilon(u) = \left(\frac{a}{b}\right)^N \left(\frac{u}{1+u}\right)^K \sum_{\ell=0}^{L-1} \frac{(K)_\ell}{\ell!\;(1+u)^\ell}\; F\!\left(N,\, K+\ell;\, K;\, \left(1 - \frac{a}{b}\right)\frac{u}{1+u}\right) . \tag{16}$$

But from (2),

$$\frac{a}{b} = \frac{1}{1 + R} , \qquad R = \frac{\bar{E}_1}{N_o} \frac{K}{N} , \tag{17}$$

where the parameters involved are described in tables 1 and 2. Making these substitutions in (16), there follows for the cumulative distribution function of random variable υ,

$$P_\upsilon(u) = \frac{1}{(1 + R)^N} \left(\frac{u}{1 + u}\right)^K \sum_{\ell=0}^{L-1} \frac{(K)_\ell}{\ell! \; (1 + u)^\ell} F\left(N, K + \ell; K; \frac{R}{1 + R} \frac{u}{1 + u}\right) . \tag{18}$$

An alternative more useful form is obtained when we use [2; 15.3.3]:

$$P_\upsilon(u) = \left(\frac{u}{1 + u}\right)^K \left(\frac{1 + u}{1 + u + R}\right)^N \sum_{\ell=0}^{L-1} \frac{(K)_\ell}{\ell!} \left(\frac{1 + R}{1 + u + R}\right)^\ell F\left(-\ell, K - N; K; \frac{R}{1 + R} \frac{u}{1 + u}\right) \tag{19}$$

for $u > 0$. This result is very attractive since the negative integer argument, $-\ell$, in the hypergeometric function causes termination of the series at ℓ terms. Thus, (19) is a closed form (albeit tedious) for the cumulative distribution function of υ, involving a finite number of elementary functions.

It should be noticed that the absolute noise level σ_n^2 does not appear in (18) or (19). (The cumulative distribution function for alternative normalizer ratio $\tilde{\upsilon}$ given by (10) can now easily be found by use of (11).)

COMPARISON WITH EARLIER RESULTS

The result (19) for the cumulative distribution function of normalizer ratio υ, operating in a partially correlated fading environment, is an approximation, having been based upon a characteristic function fitting procedure explained in [1; (A-24)-(A-28)]. Nevertheless, (19) is identical with the exact fading result for a related normalizer problem; namely, agreement with [4; (25)] is achieved under the following identifications:

TR 4783	Here	Interpretation
α	u	threshold
M	K	number of signal pulses
N	L	number of noise-only pulses
$\upsilon + 1$	N	m K_e, table 2
μ	R	$\frac{\overline{E}_1}{N_0}\frac{K}{N}$, (2)

Table 3. Identification of Variables

The identity of $\upsilon + 1$ with N is made by comparing [4; (24A)] with [1; (A-29)]. The final identity of μ with R utilizes [4; (24B)] and [1; (9)]:

$$\mu = \frac{\overline{R}_T}{\upsilon + 1} \rightarrow \frac{\overline{E}_T/N_0}{N} = \frac{\overline{E}_1 \; K/N_0}{N} = R \; , \qquad (20)$$

where the arrow indicates transferrance from [4] to [1].

The approach in [4] proceeded as follows: the detection probability for nonfading signals in all the bins depended only on the total received signal-to-noise ratio R_T. When R_T was assigned the fading probability density function [4; (24A)], the average detection probability in [4; (25)] resulted. For the special case of fading parameter $\upsilon = M - 1$ there, numerous graphical results were given in [4; figures 1-36].

The current results here are more general, in that they allow for partially correlated fading (through parameter K_e) and a more general power-fading model (with 2m degrees of freedom). This means that $N = m\, K_e$ here is not restricted to be equal to the number of signal pulses, K, but is arbitrary. Thus the current numerical results will significantly augment and extend those in [4]. If N = K here, then $R = \overline{E_1}/N_0$ = signal-to-noise ratio per pulse, and (19) reduces to [4; (15B)], for which many numerical results were given in [4; figures 1-36].

SPECIAL CASES

For m = 1, which corresponds to Rayleigh amplitude fading, and for $\rho_{kj} = \delta_{kj}$, which corresponds to uncorrelated fading, then $K_e = K$, N = K, and we get from (19),

$$P_\upsilon(u) = \left(\frac{u}{1 + u + R}\right)^K \sum_{\ell=0}^{L-1} \frac{(K)_\ell}{\ell!} \left(\frac{1 + R}{1 + u + R}\right)^\ell , \tag{21}$$

in agreement with [4; (15B)].

On the other hand, if R = 0, then (18) and (19) both reduce to

$$P_v^{(o)}(u) = \left(\frac{u}{1+u}\right)^K \sum_{\ell=0}^{L-1} \frac{(K)_\ell}{\ell!\,(1+u)^\ell} , \tag{22}$$

which is equal to $1 - P_F$, where P_F is the false alarm probability. Since noise level σ_n^2 is not involved in (22), threshold u can be selected to realize a given P_F, once K and L have been specified. This is a quantitative verification of the expected constant false alarm rate property of the normalizer.

Finally, in the special case of one signal pulse, K = 1, and Rayleigh amplitude fading, m = 1, then $K_e = 1$, N = 1, $R = \overline{E}_1/N_o$, and (19) yields

$$P_v(u) = \frac{u}{1+u+R} \sum_{\ell=0}^{L-1} \left(\frac{1+R}{1+u+R}\right)^\ell = 1 - \left(\frac{1+R}{1+u+R}\right)^L . \tag{23}$$

That is,

$$1 - P_v(u) = \left(1 + \frac{u}{1+R}\right)^{-L} , \tag{24}$$

which agrees with [5; (6)] when we make the identifications (from there to here) of $N \to L$, $T/N \to u$, $\gamma \to R$.

RECURSION FOR CUMULATIVE DISTRIBUTION FUNCTION

Let the hypergeometric function appearing in (19) be represented as follows:

$$H_\ell(x) \equiv \frac{(K)_\ell}{\ell!} F(-\ell, K-N; K; x) . \tag{25}$$

Then

$$H_0(x) = 1 \ , \tag{26}$$

while (25) has the recursion [2; 15.2.10]

$$\ell H_\ell(x) = [K + 2\ell - 2 + (N - K + 1 - \ell)x]\ H_{\ell-1}(x) - (K + \ell - 2)(1 - x)H_{\ell-2}(x) \quad \text{for } \ell \geq 1 \ , \tag{27}$$

where we define $H_{-1}(x) = 0$. In terms of (25), the cumulative distribution function of v in (19) becomes

$$P_v(u) = \left(\frac{u}{1 + u}\right)^K \left(\frac{1 + u}{1 + u + R}\right)^N \sum_{\ell=0}^{L-1} \left(\frac{1 + R}{1 + u + R}\right)^\ell H_\ell\left(\frac{R}{1 + R}\ \frac{u}{1 + u}\right) . \tag{28}$$

This form, in conjunction with recursion (27), was used for all the numerical results here, for L finite. The parameters appearing in (28) have all been explained in tables 1 and 2. The explicit dependence on the fundamental parameters is indicated below:

$$K_e = K_e(K, \{\rho_{kj}\}) \ ,$$
$$N = N(m, K, \{\rho_{kj}\}) \ ,$$
$$R = R(\overline{E_1}/N_0, m, K, \{\rho_{kj}\}) \ . \tag{29}$$

In addition, the cumulative distribution function in (28) is a function of L and threshold u.

DETECTION AND FALSE ALARM PROBABILITIES

The detection probability is given by

$$P_D = \mathrm{Prob}(v > u \mid R > 0) = 1 - P_v(u) , \tag{30}$$

where $P_v(u)$ is available in (28). The false alarm probability is

$$P_F = \mathrm{Prob}(v > u \mid R = 0) = 1 - P_v^{(o)}(u) , \tag{31}$$

where $P_v^{(o)}(u)$ is available in (22). By allowing threshold u to vary over a wide range, P_D and P_F values can be obtained and plotted against each other, resulting in the standard receiver operating characteristics; the threshold is thereby eliminated from the plotted outputs. Programs for plotting P_D vs P_F, both for L finite as well as infinite, are listed in appendix A.

GRAPHICAL RESULTS

Due to the multitude of parameters appearing in this investigation (see tables 1 and 2), it is impossible to give a comprehensive compilation of encompassing numerical results. Considering just the covariance coefficients $\{\rho_{kj}\}_1^K$ for the moment, complete specification requires assignment of K(K - 1)/2 values to these quantities; to circumvent this difficulty, we consider numerically, here, only the very special case of exponential correlation, for which

$$\rho_{kj} = \rho^{|k-j|} \quad \text{for} \quad 1 \le k,\ j \le K\ , \tag{32}$$

and look at a couple of particular values for ρ. Our approach here, of necessity, is to give some representative sample receiver operating characteristics and a general computer program in BASIC, whereby additional results can easily by obtained once the user has specified all the particular values of interest in his application. This program allows for arbitrary covariance coefficients, $\{\rho_{kj}\}$, and is not limited to the specific example (32).

The particular cases we will investigate are as follows:

$$\begin{aligned} K &= 1,\ 2,\ 4, \\ L &= 16,\ 32,\ \infty, \\ m &= .5,\ 1, \\ \rho &= 0,\ .5\ . \end{aligned} \tag{33}$$

All possible combinations of these four fundamental variables lead to 30 plots, which appear below* in figures 2-31. (There are only 6 plots for K = 1, not 12, because the value of ρ is irrelevant for K = 1). The curves are indexed by the per-pulse signal-to-noise ratio, $\overline{E}_1/N_0$, in dB. The false alarm and detection probability pairs range from (poor quality) pair (.5,.01) up to (high quality) pairs near (1E-10,.999).

The number of signal pulses, K, is limited to the low values 1, 2, 4, because these seem to be the cases of most immediate practical use. The number of noise-only samples, L, is not evaluated for L = 64 because of the proximity of the results to those for $L = \infty$; conversely, results are not presented for L = 8, because a severe degradation in performance occurs, that probably cannot be tolerated. The fading parameter value m = 1 corresponds to Rayleigh amplitude fading (exponential power fading), while m = .5 corresponds to a deeper more-damaging form of fading. The correlation coefficient $\rho = 0$ corresponds to uncorrelated (independent) fading, while $\rho = .5$ allows for adjacent (equispaced) pulses in figure 1 to have some degree of dependent fading.

An explanation of the initial result in figure 2 follows: for K = 1, m = 1, $L = \infty$ (known noise level), the detection probability is plotted versus the false alarm probability for values of the latter between 1E-10 and .1. The value of the per-pulse signal-to-noise ratio, $\overline{E}_1/N_0$ in dB, varies over the range 6, 8, 10, ... , 42, giving detection probability values covering

* All the figures are collected together after the Summary section.

the range .01 to .999. The only difference in the accompanying pair, figures 3 and 4, is that L is reduced to 32 and 16, respectively.

The results in figures 5 through 7 correspond to the worst cases considered here. Namely, there is just one (fading) signal pulse, and m is .5, which means a very deep fading medium; see [1; figure 2]. The values of signal-to-noise ratio required for L = 16 in figure 7 are so large as to be physically unrealistic, except for the poorer quality region.

On the other hand, for K = 4 signal pulses, Rayleigh amplitude fading (m = 1), and uncorrelated fading (ρ = 0), the results in figures 20 through 22 are very encouraging, being physically reasonable over the whole range of plotted values. But when m is decreased to .5, and ρ is increased to .5, the results in figures 29 through 31, still for K = 4 pulses, indicate substantially increased signal-to-noise ratio requirements at the higher quality end of the performance region.

An alternative method of presenting the graphical results, which accounts for the losses incurred by not knowing the noise level, is to plot the required value of $\overline{E}_1/N_0$ vs L, for various values of the remaining parameters and for specified performance quality in terms of P_F and P_D. Two such cases are illustrated in figures 32 and 33. They show that the cost of not knowing the noise level is not severe for the high false alarm probabilities, but is quite significant for the lower more-desirable false alarm probabilities. For example, in figure 33 for K = 2 signal pulses, the signal-to-noise ratio must be about 1.5 dB larger at L = 10 noise pulses than

at L = 100, when P_F = .01. However, if we want to operate at P_F = 1E-10, the increased signal-to-noise ratio requirement is about 6 dB per pulse. The numbers are comparable for the K = 1 results in figure 32.

The asymptotes for large L in figures 32 and 33 can be found in some cases from earlier results in [1]. For example, reference to [1; figure 8] for K = 2, ρ = .5 gives $\overline{E}_1/N_0 \cong$ 16.8 dB, while P_F = 1E-6, P_D = .9, m = 1. Comparison with figure 33 here reveals that the performance requirement is virtually at this level by the time that L = 100.

SUMMARY

Although figures 32 and 33 are very informative, allowing for a ready assessment of the losses incurred by using a finite small value for L, the number of noise-only pulses, they also illustrate the voluminous compilation that would be needed for a thorough numerical investigation. For example, if: detection probabilities P_D were of interest for values .5, .9, .99, .999; number of signal pulses K for values 1, 2, ... , 10; fading parameter m for values .5, 1, 2; and fading correlation coefficient ρ for 0, .5, 1; this would require a total of 4*10*3*3 = 360 figures. The approach here is instead to present some representative receiver operating characteristics, in figures 2 through 31, from which information similar to that in figures 32 and 33 can be extracted, and to list a general program for the generation of additional receiver operating characteristics for whatever cases may be of interest to the user.

Some related work on the performance of a log-normalizer subject to Weibull or log-normal inputs has been published by the author in [6]; however, no fading was allowed, and the number of signal pulses was limited to K = 1. In a different vein, the performance of an or-ing device operating on the output of an incoherent combiner of multiple pulses was analyzed in [7]. These works augment and complement the analysis conducted here.

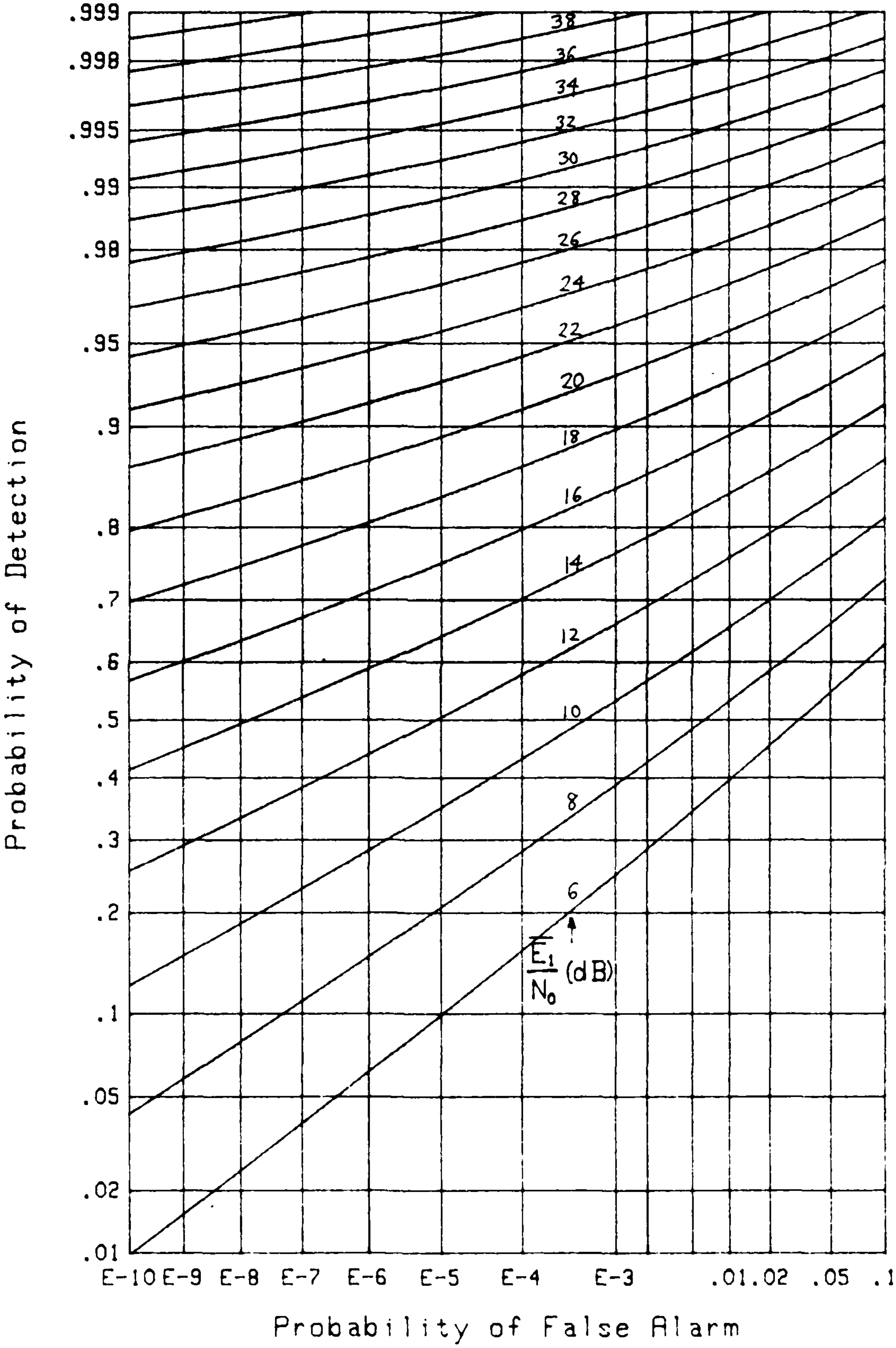

Figure 2. ROC for K=1, m=1, L=∞

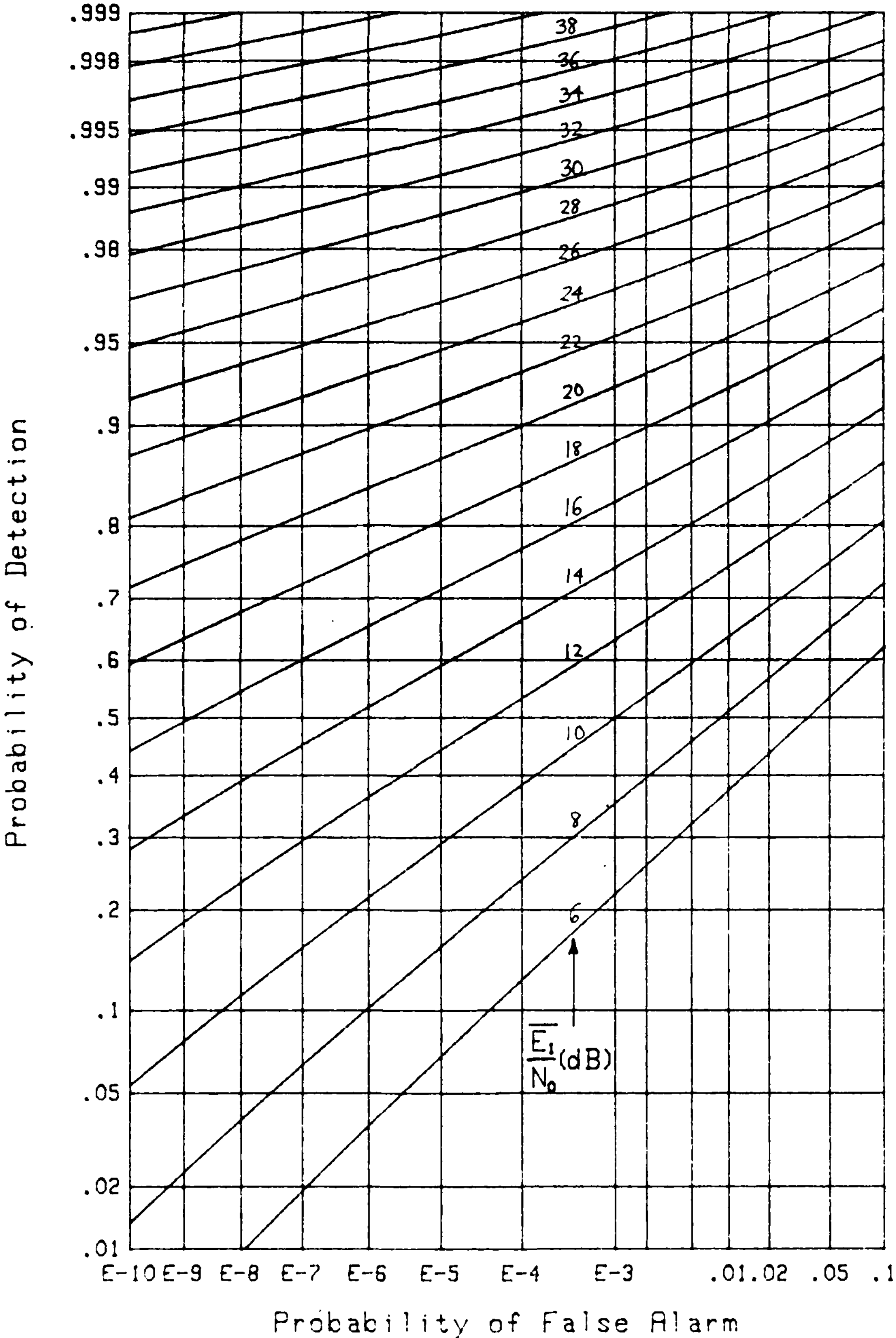

Figure 3. ROC for K=1, m=1, L=32

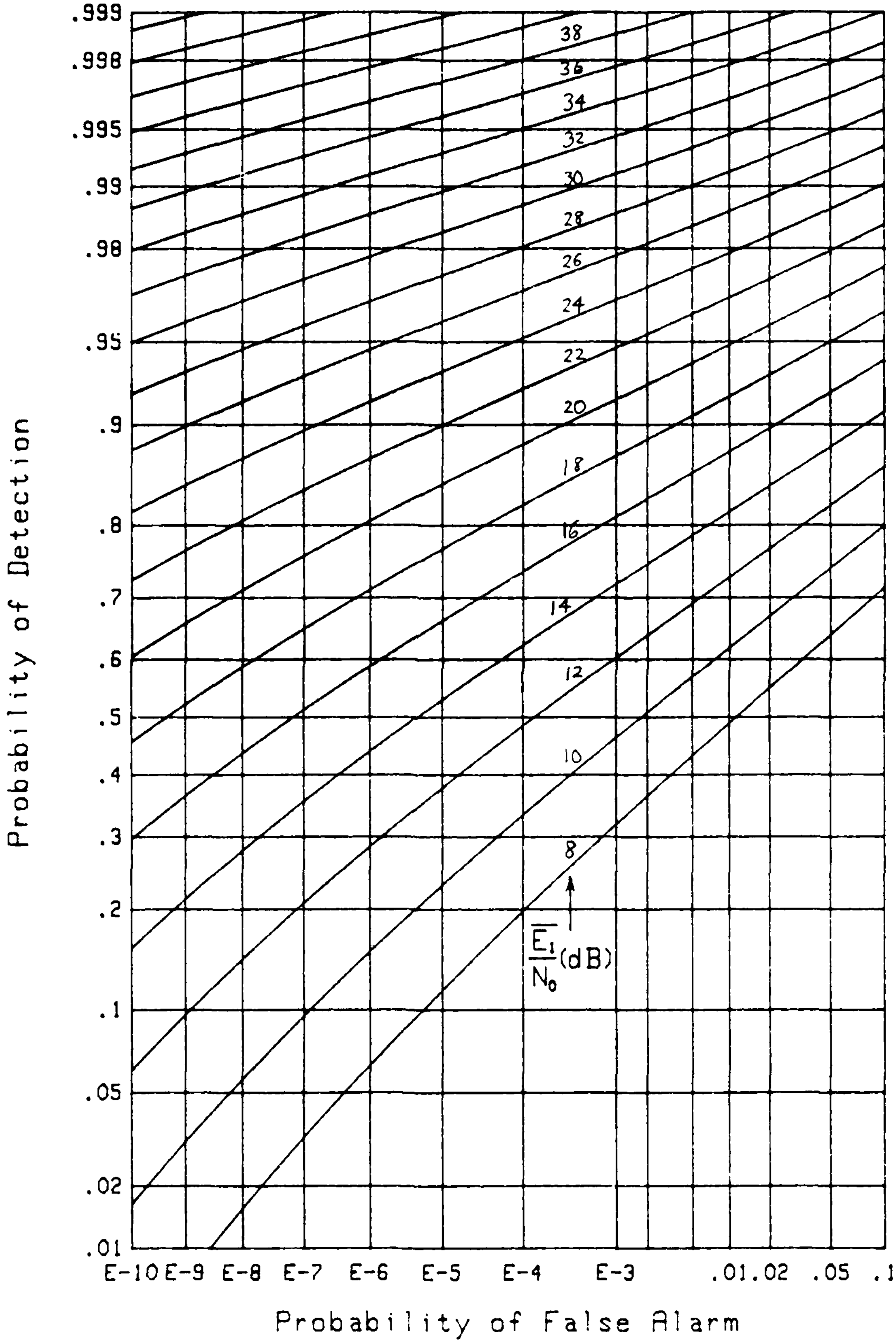

Figure 4. ROC for K=1, m=1, L=16

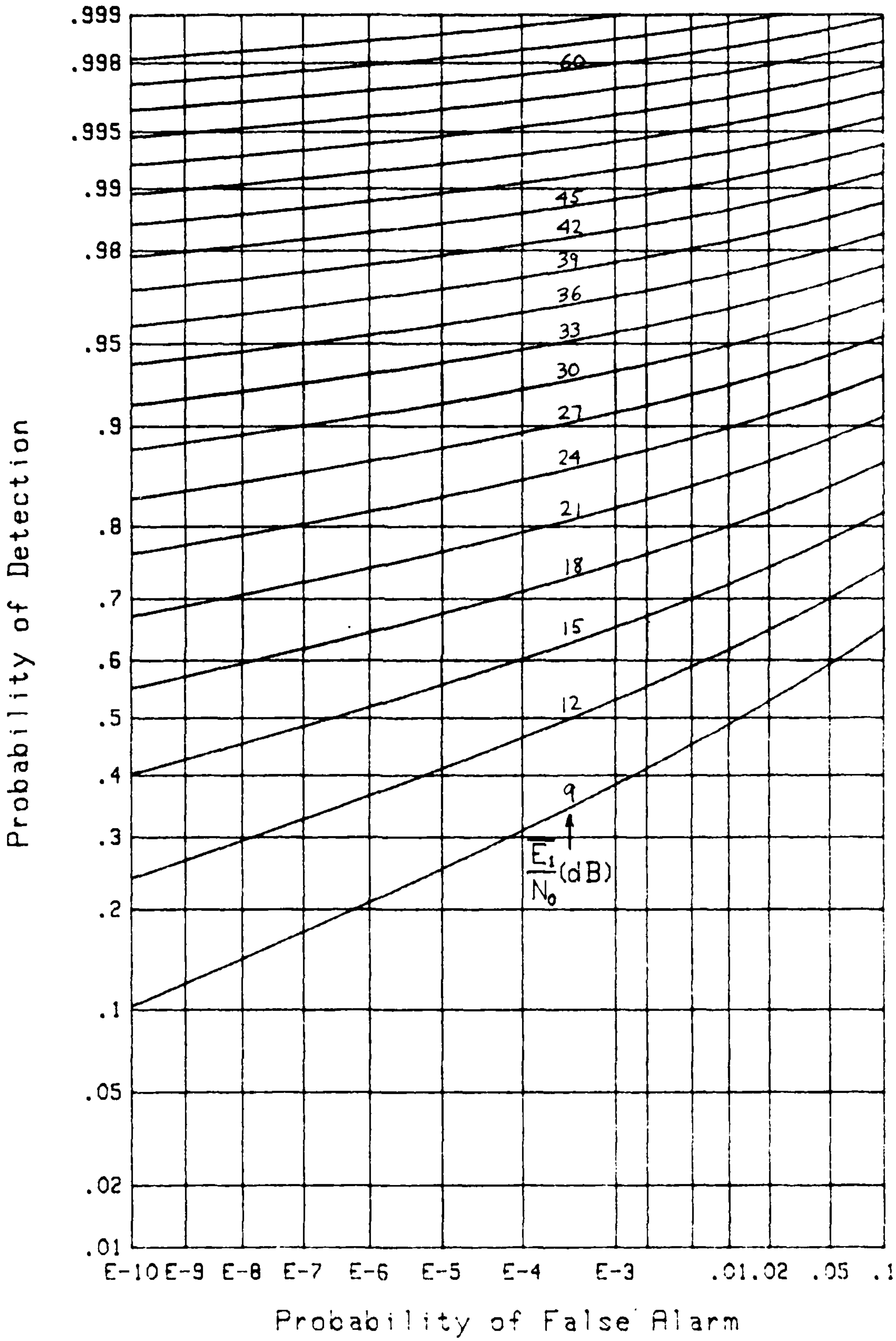

Figure 5. ROC for K=1, m=.5, L=∞

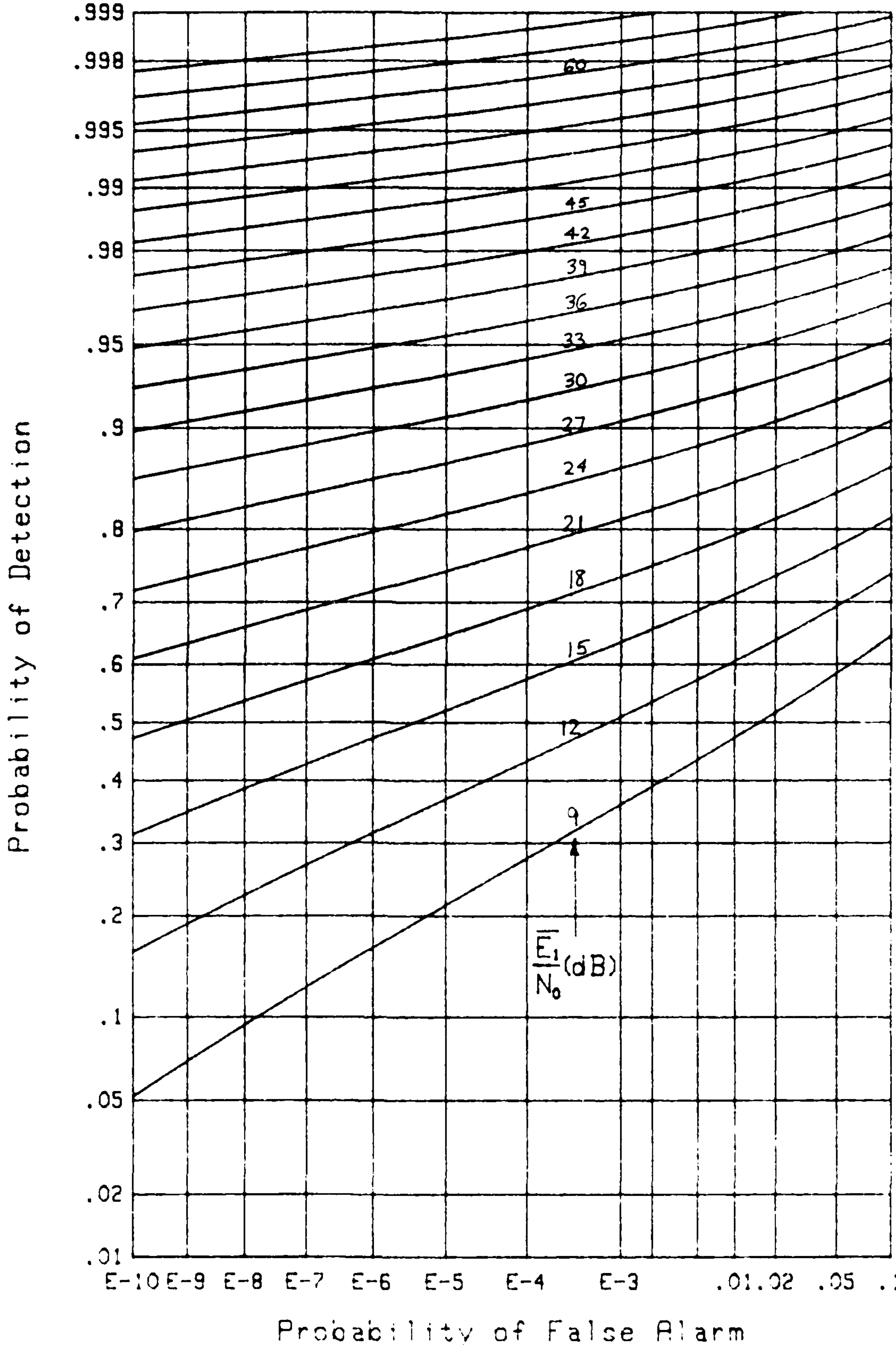

Figure 6. ROC for K=1, m=.5, L=32

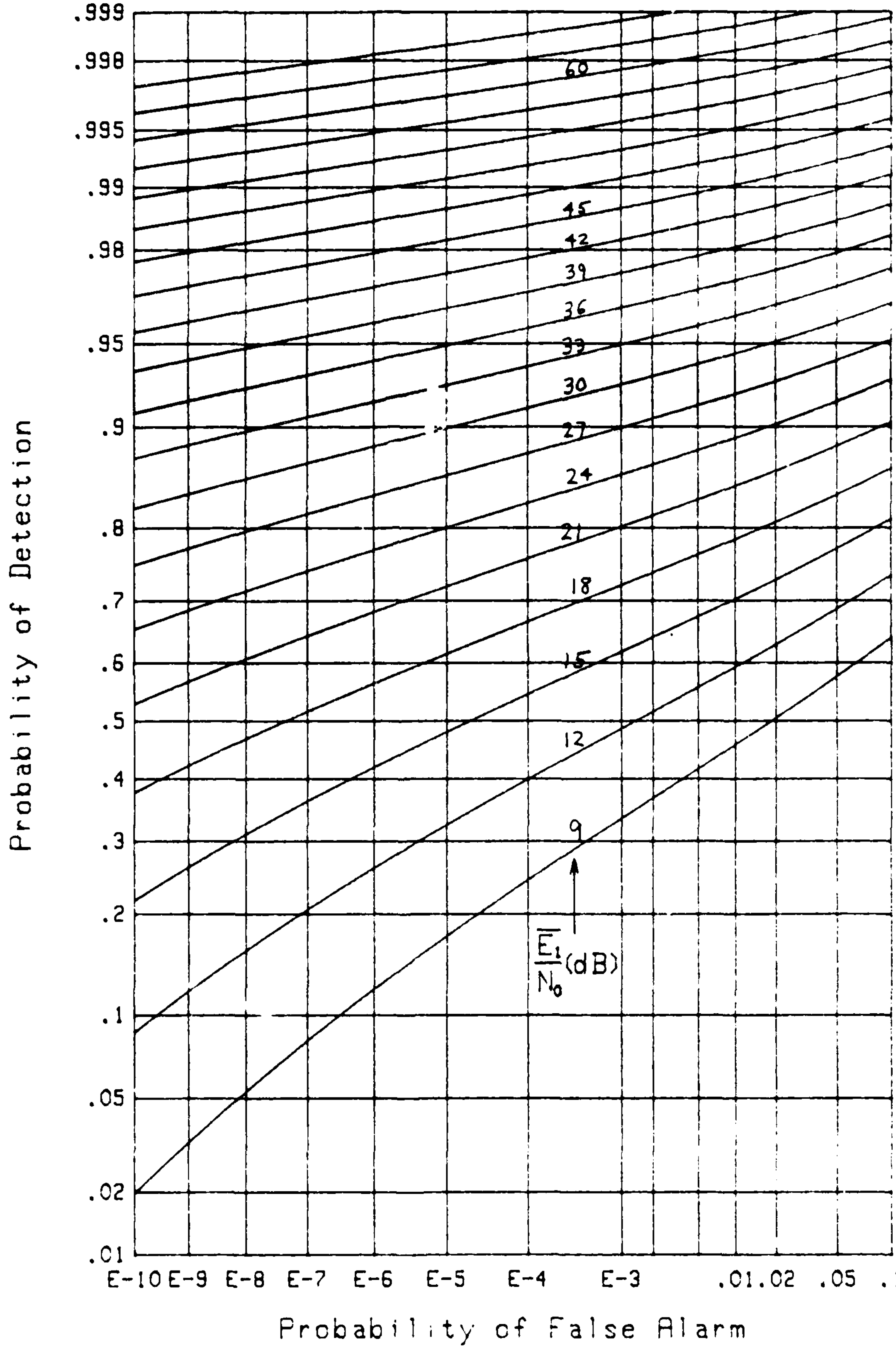

Figure 7. ROC for K=1, m=.5, L=16

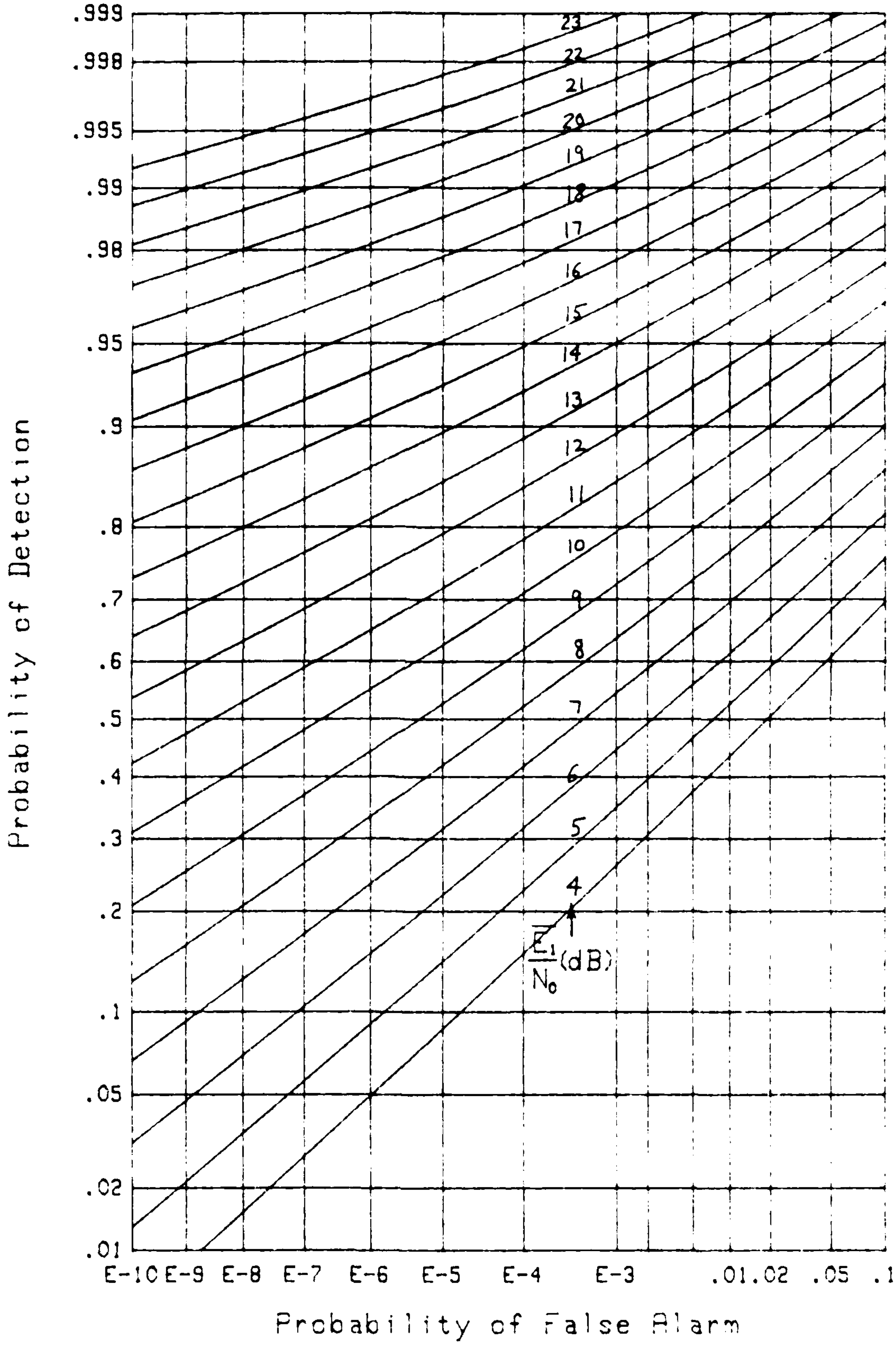

Figure 8. ROC for K=2, m=1, ρ=0, L=∞

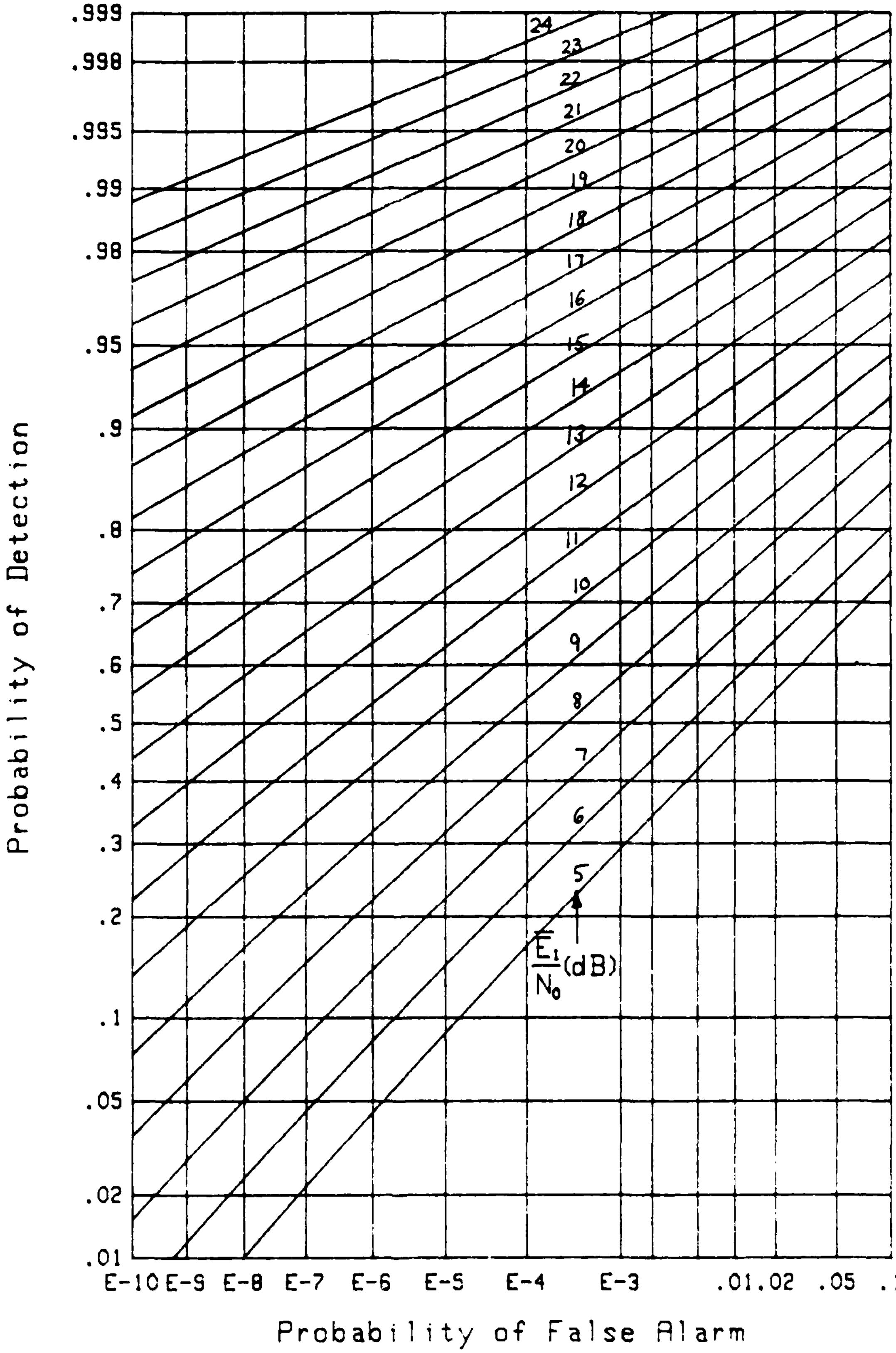

Figure 9. ROC for K=2, m=1, ρ=0, L=32

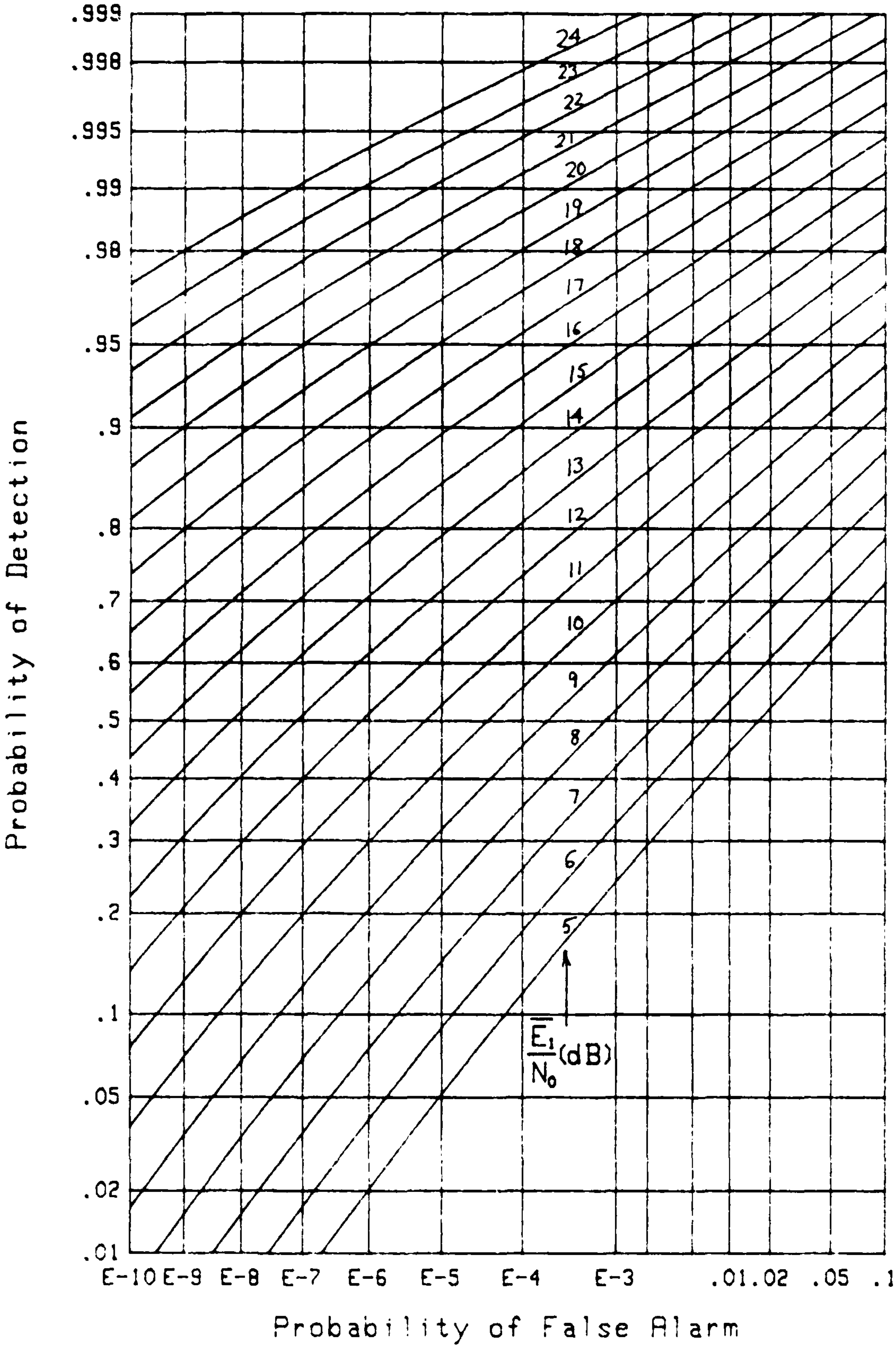

Figure 10. ROC for K=2, m=1, ρ=0, L=16

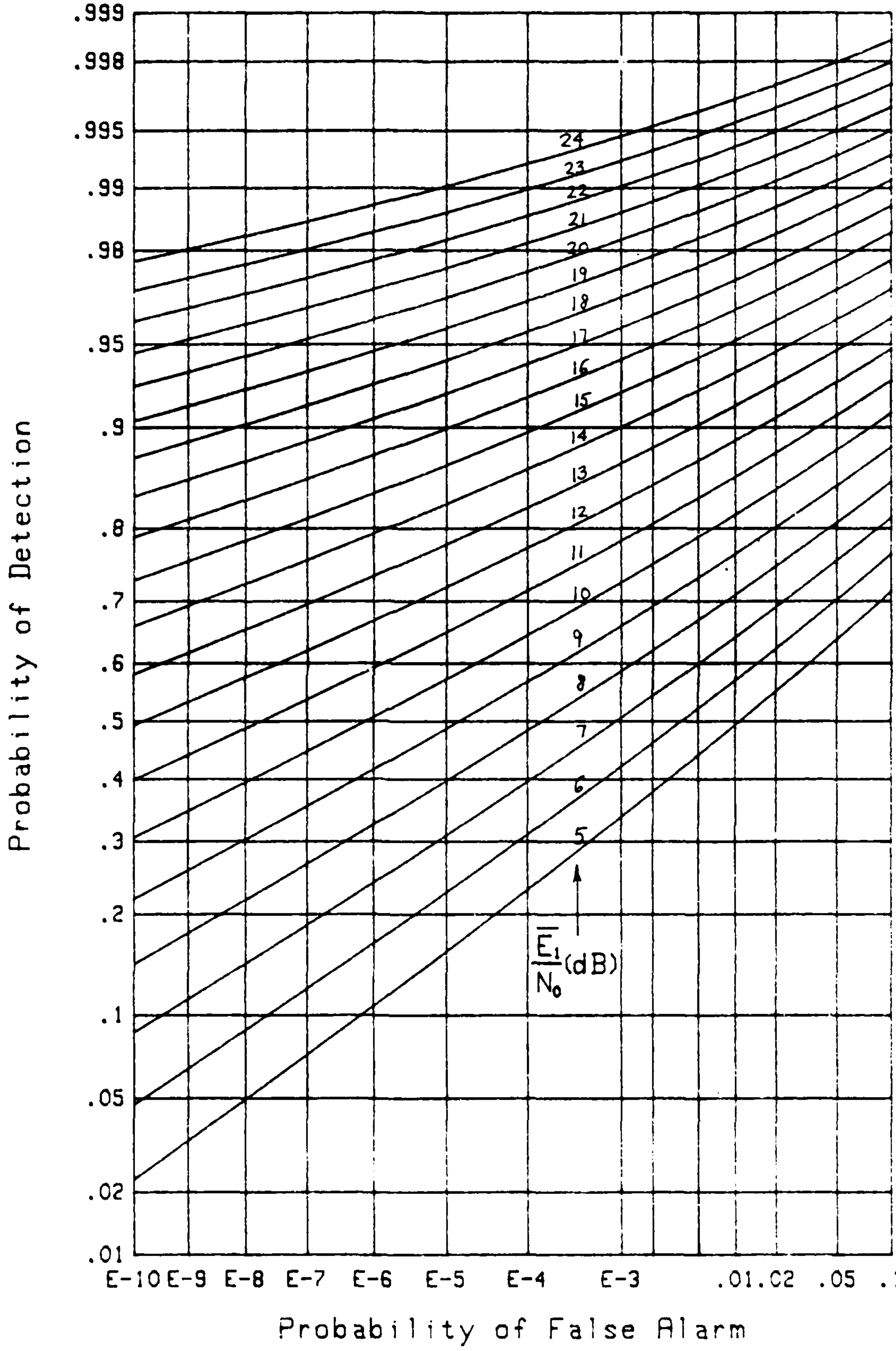

Figure 11. ROC for K=2, m=1, ρ=.5, L=∞

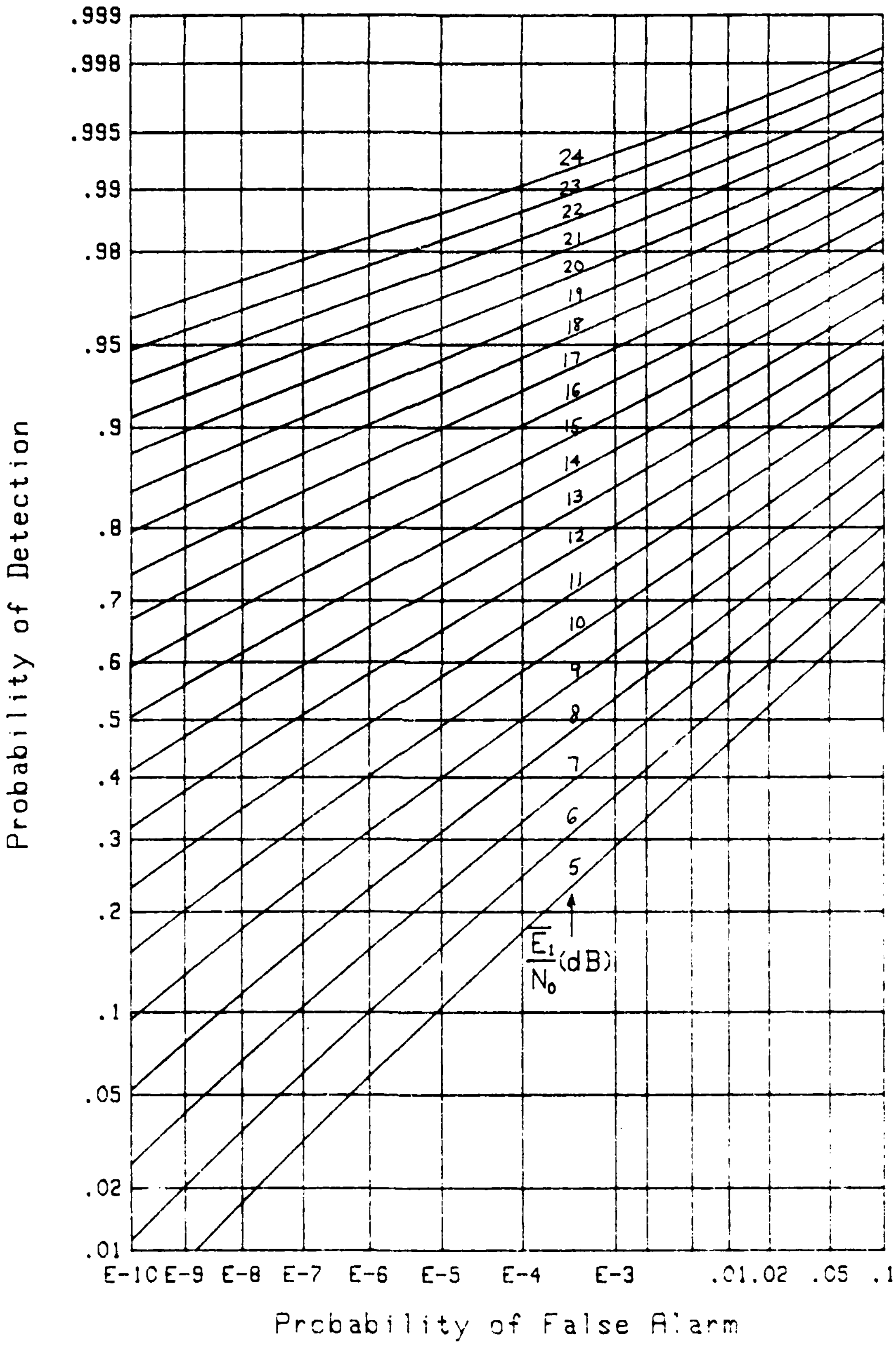

Figure 12. ROC for K=2, m=1, ρ=.5, L=32

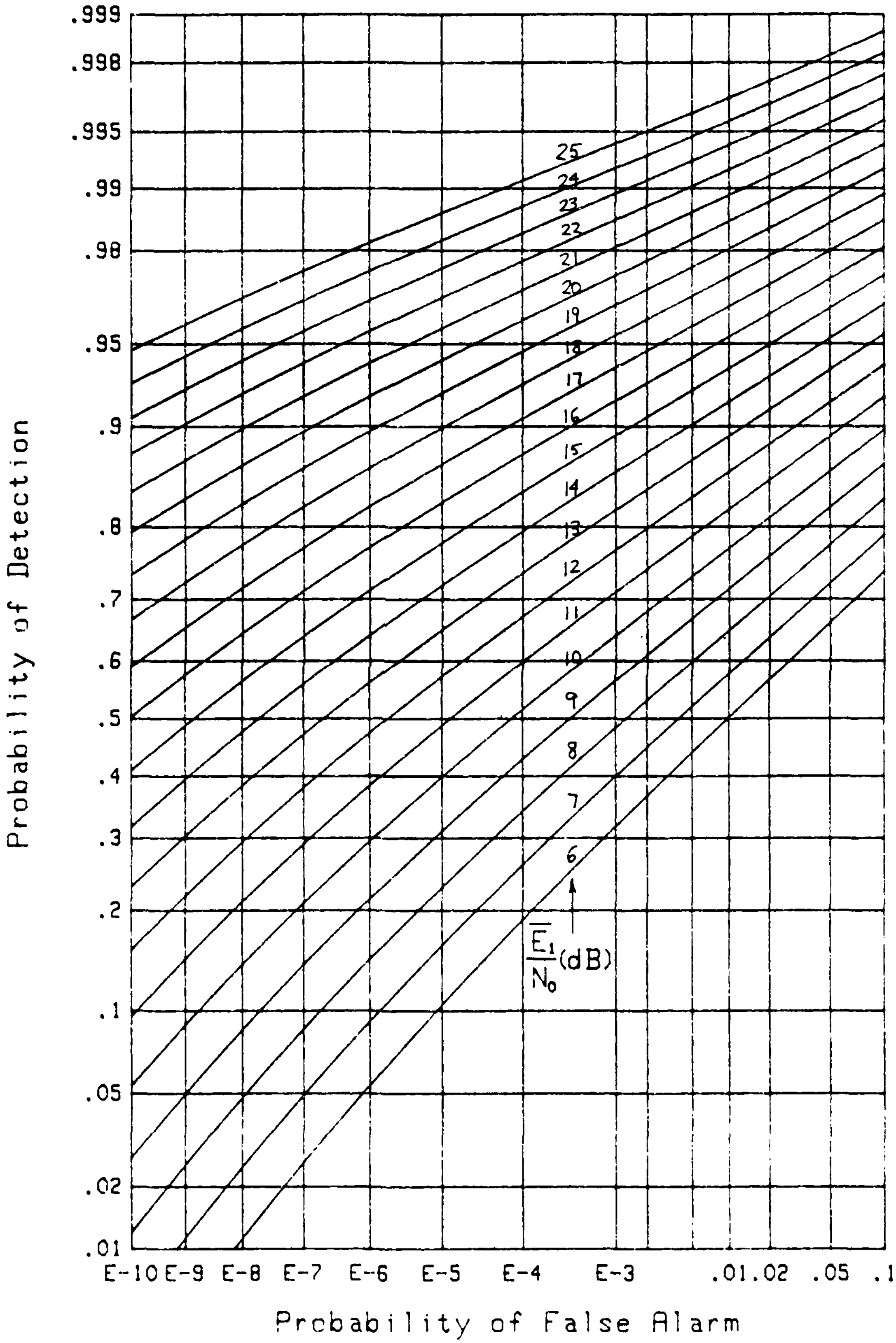

Figure 13. ROC for K=2, m=1, ρ=.5, L=16

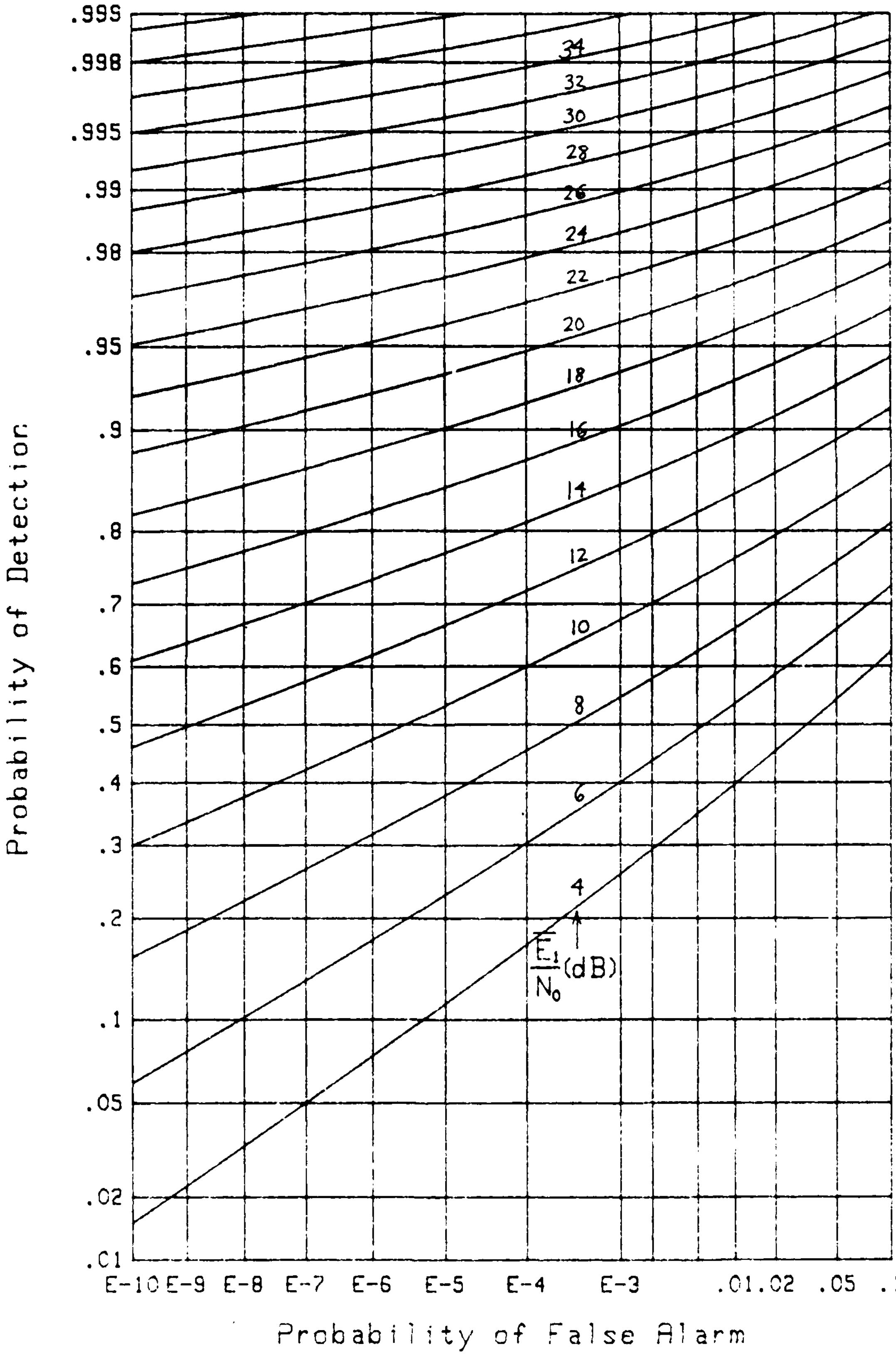

Figure 14. ROC for K=2, m=.5, ρ=0, L=∞

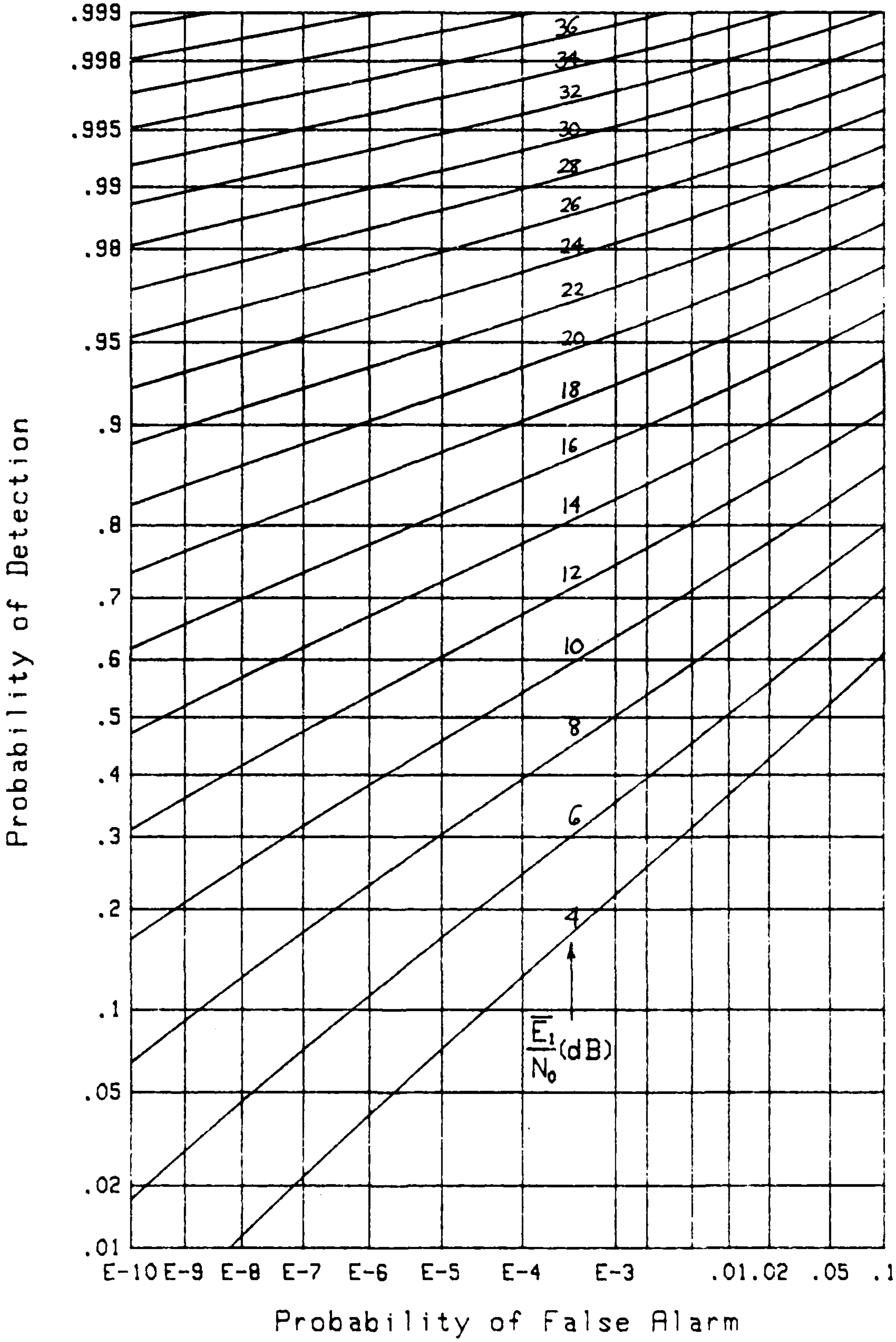

Figure 15. ROC for K=2, m=.5, ρ=0, L=32

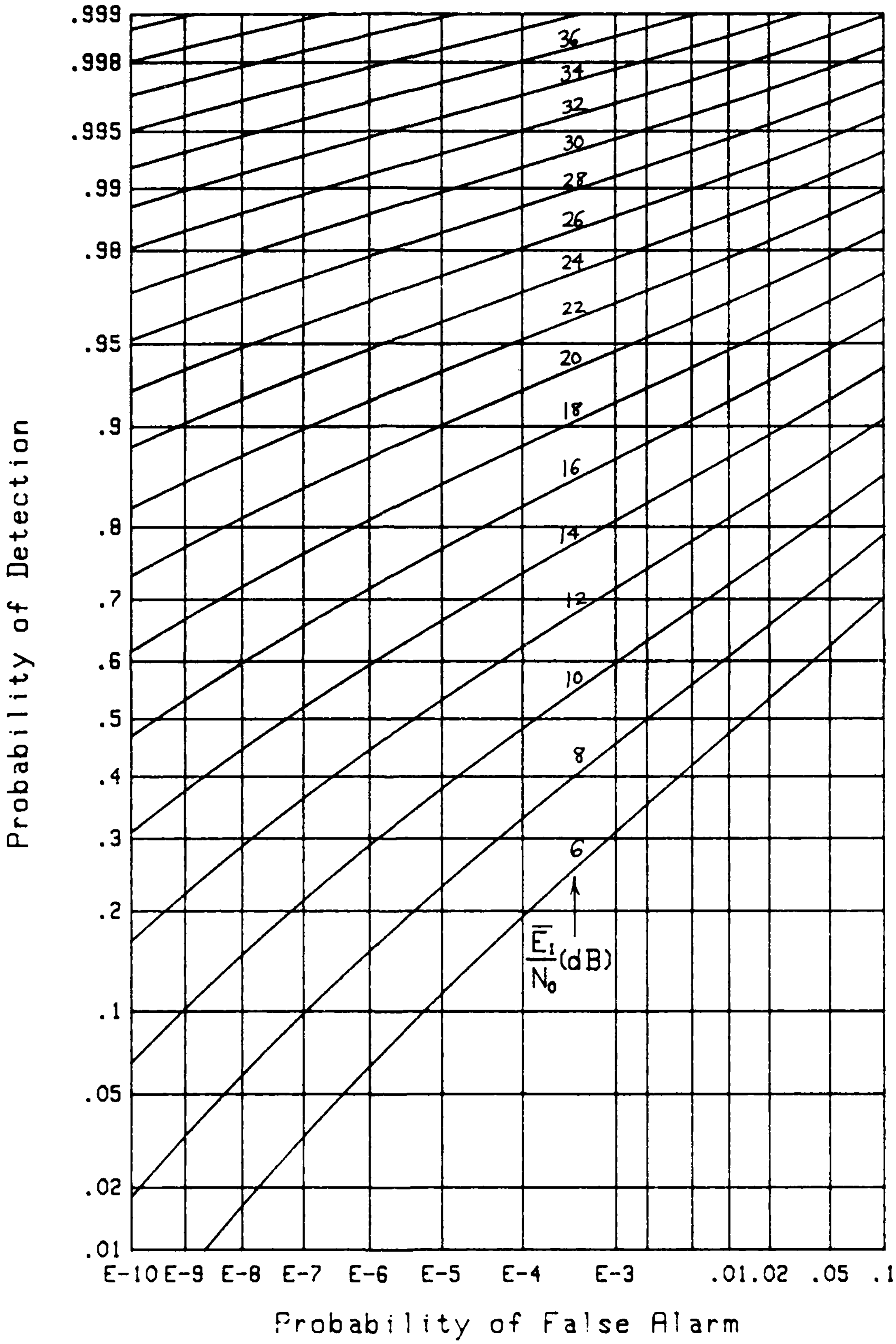

Figure 16. ROC for K=2, m=.5, ρ=0, L=16

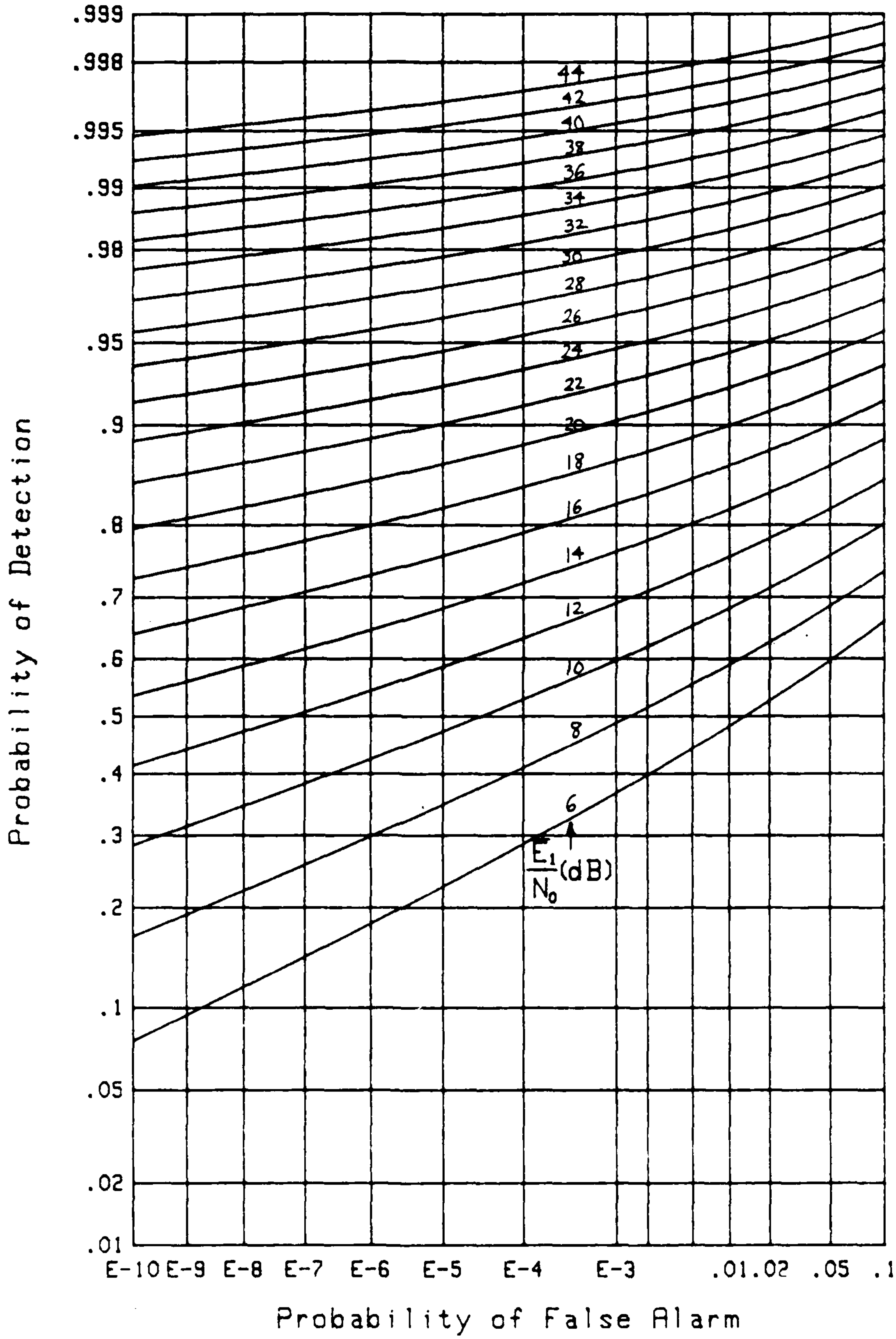

Figure 17. ROC for K=2, m=.5, ρ=.5, L=∞

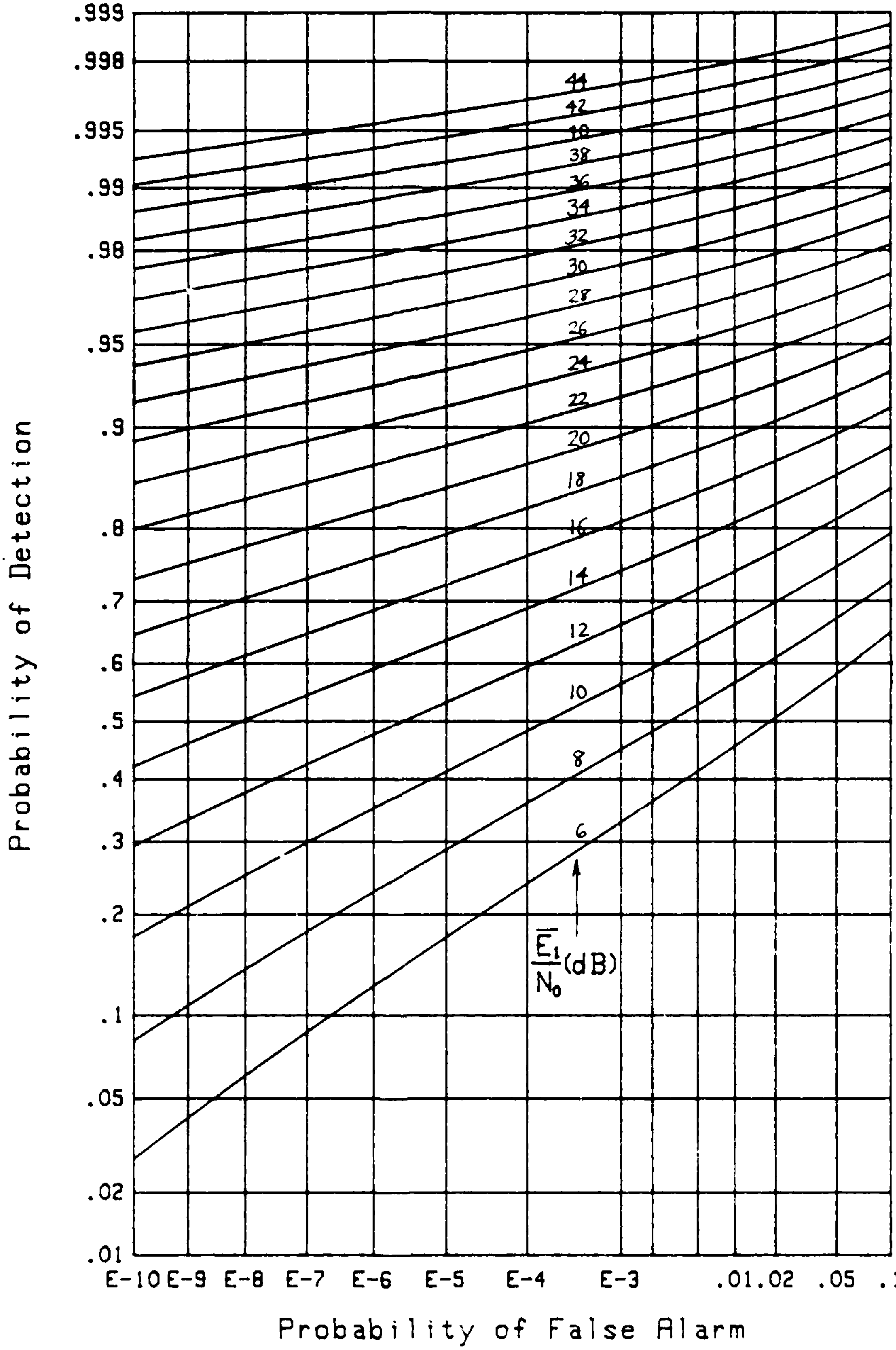

Figure 18. ROC for K=2, m=.5, ρ=.5, L=32

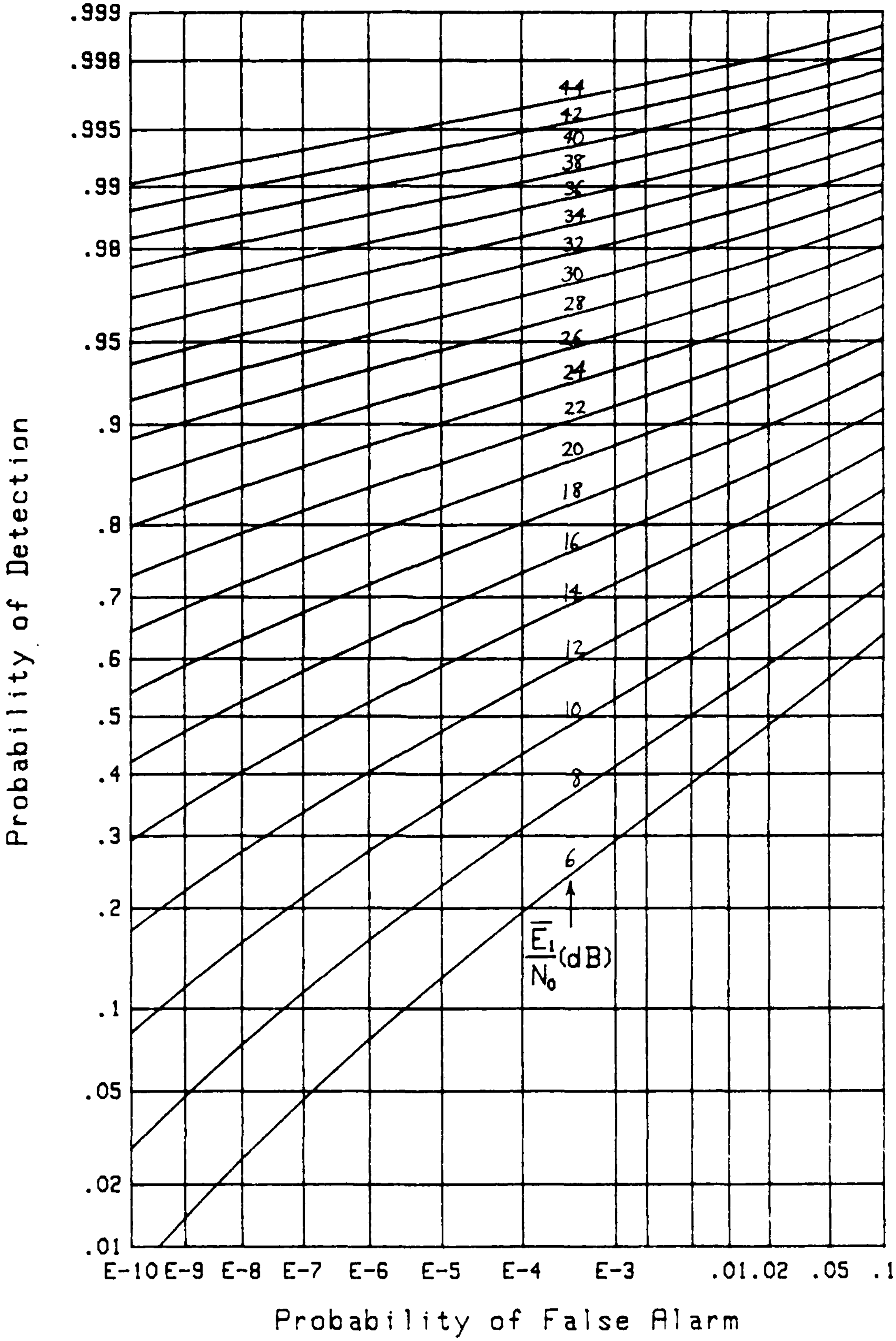

Figure 19. ROC for K=2, m=.5, ρ=.5, L=16

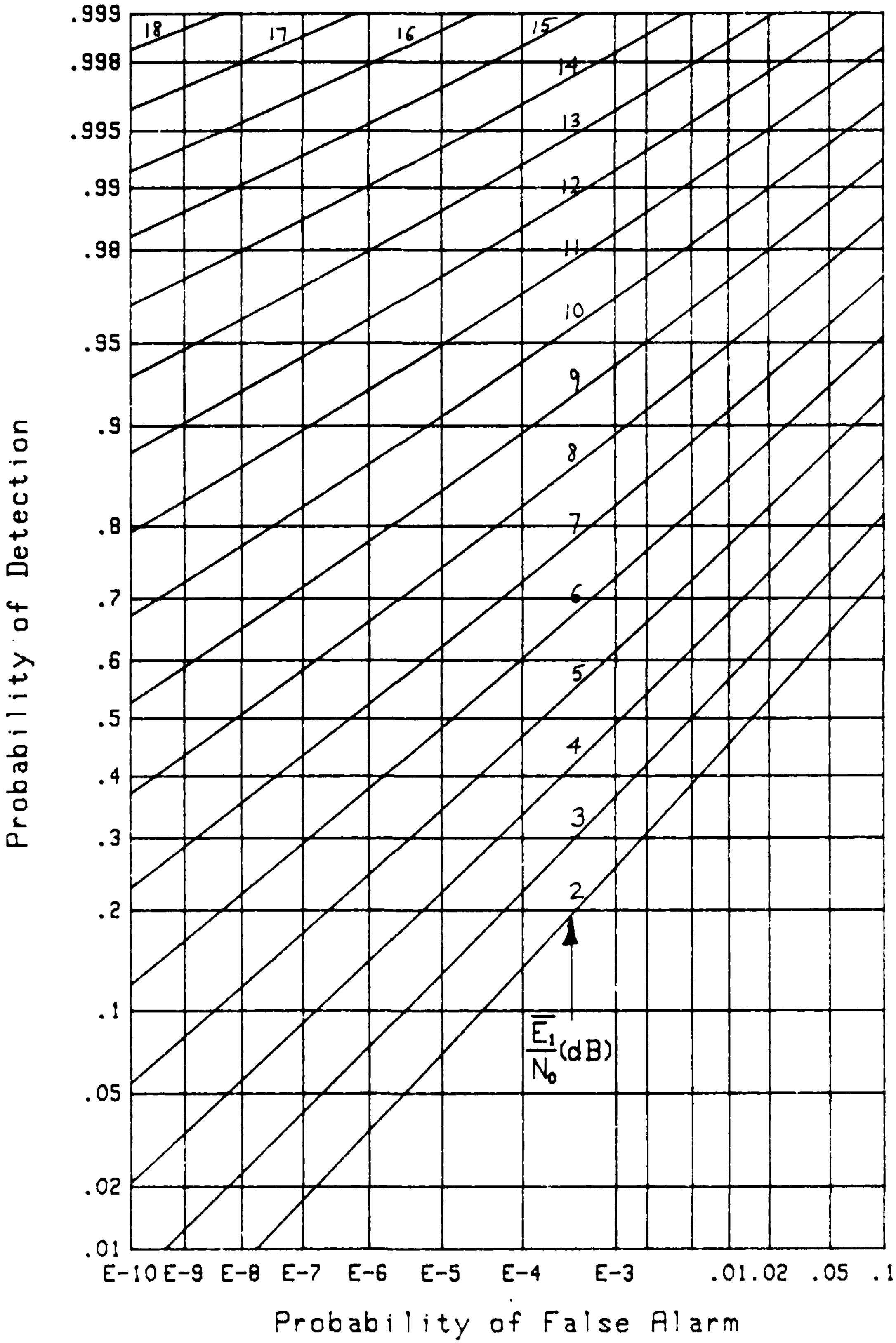

Figure 20. ROC for K=4, m=1, ρ=0, L=∞

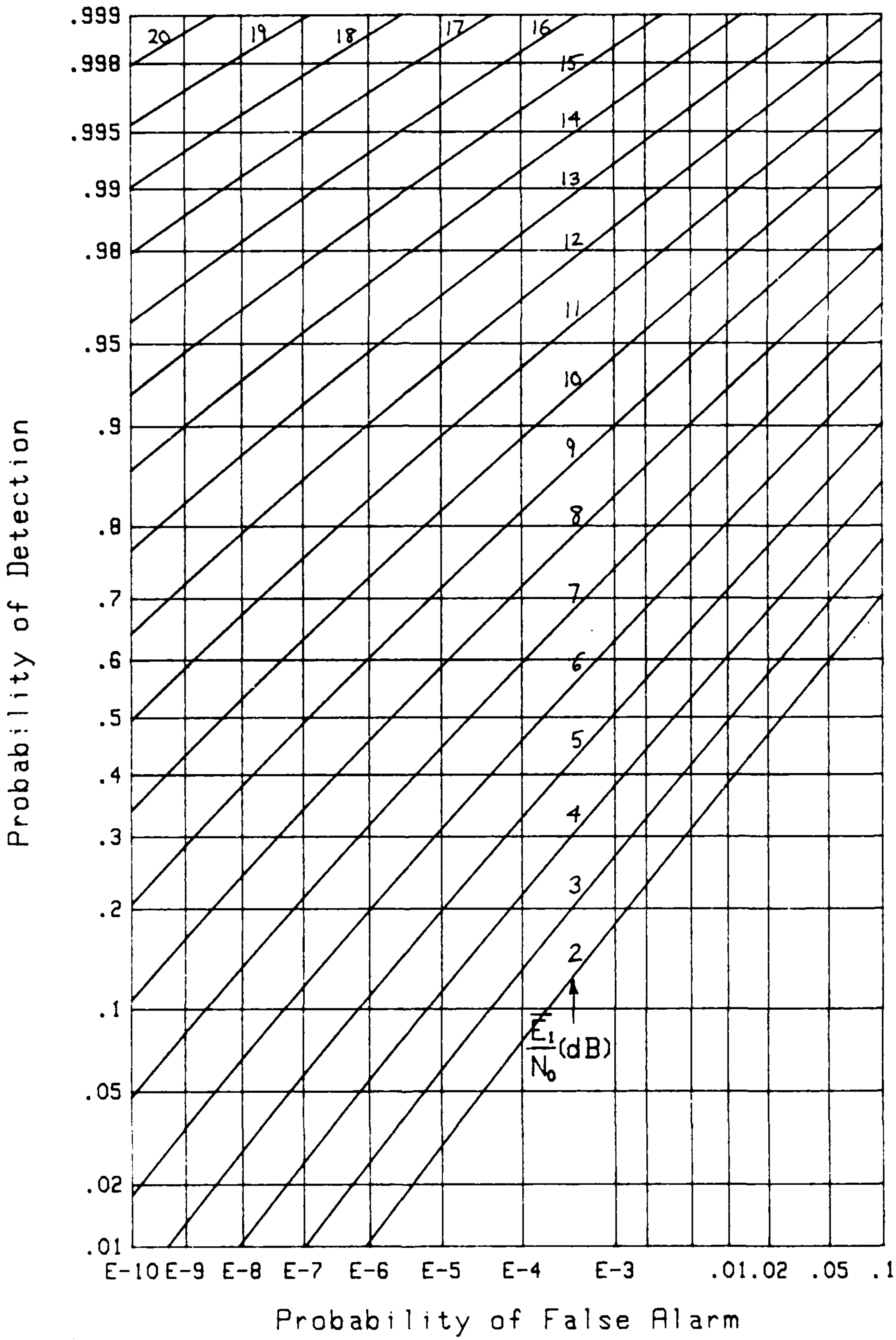

Figure 21. ROC for K=4, m=1, ρ=0, L=32

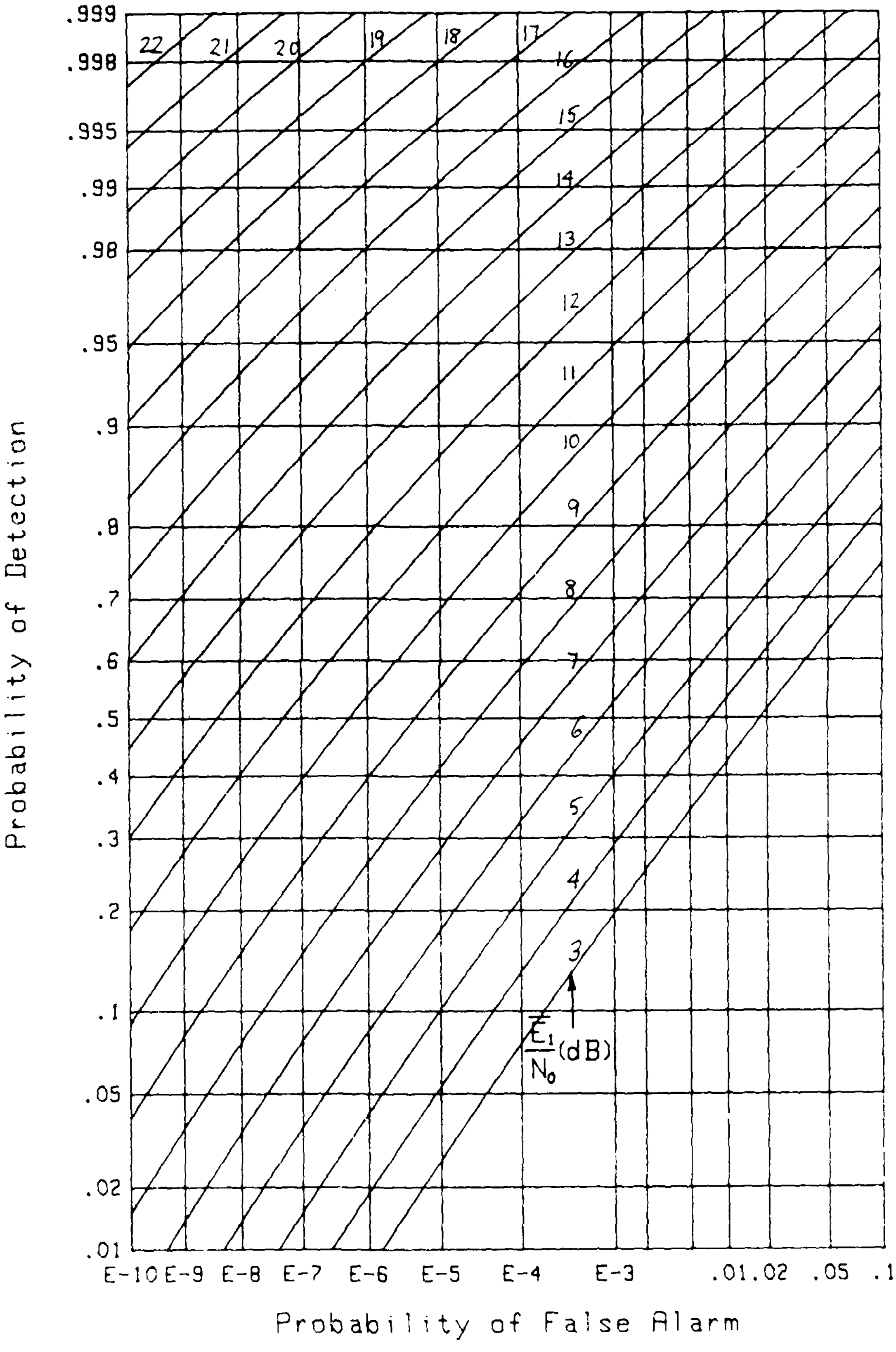

Figure 22. ROC for K=4, m=1, ρ=0, L=16

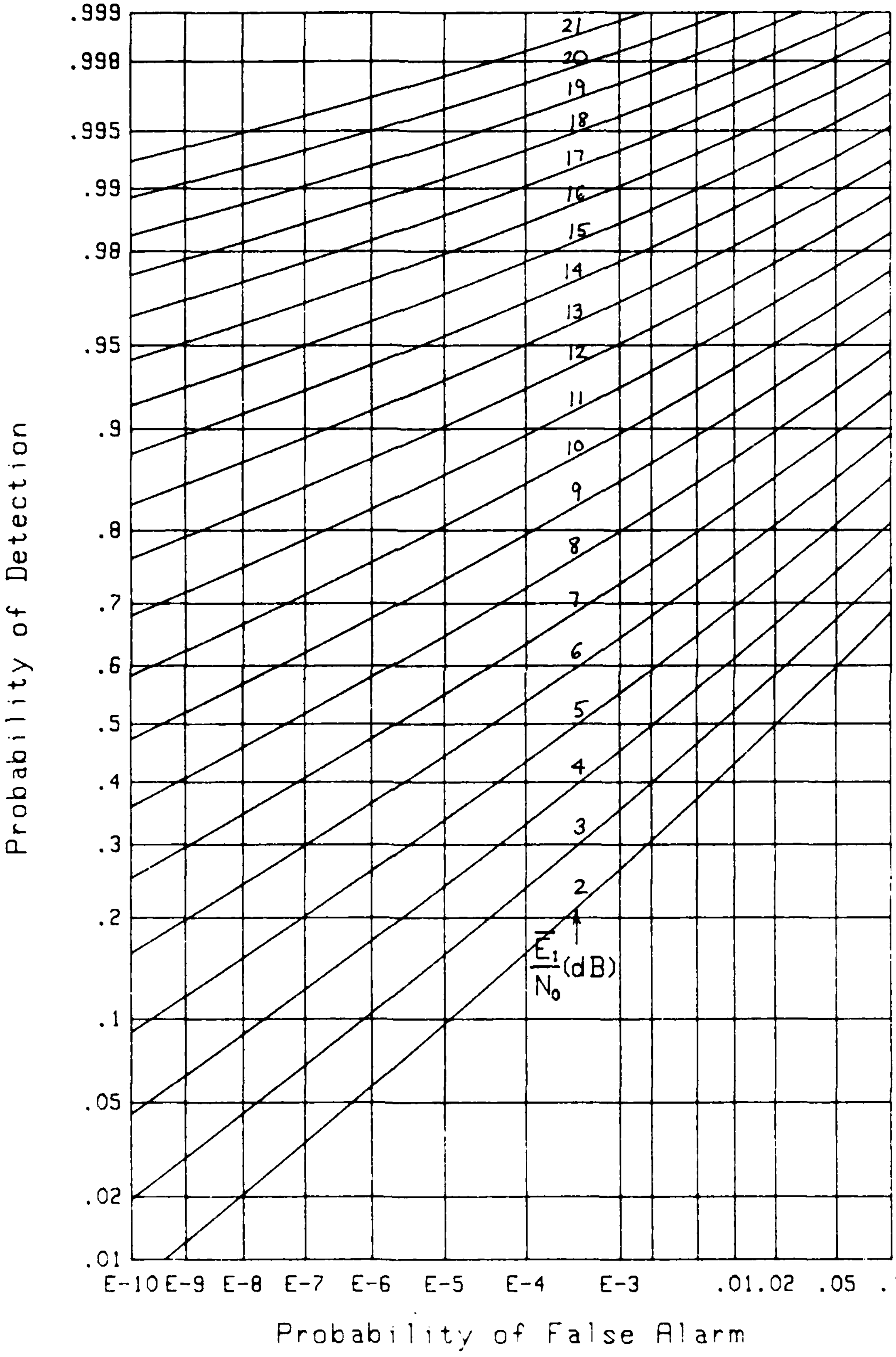

Figure 23. ROC for K=4, m=1, ρ=.5, L=∞

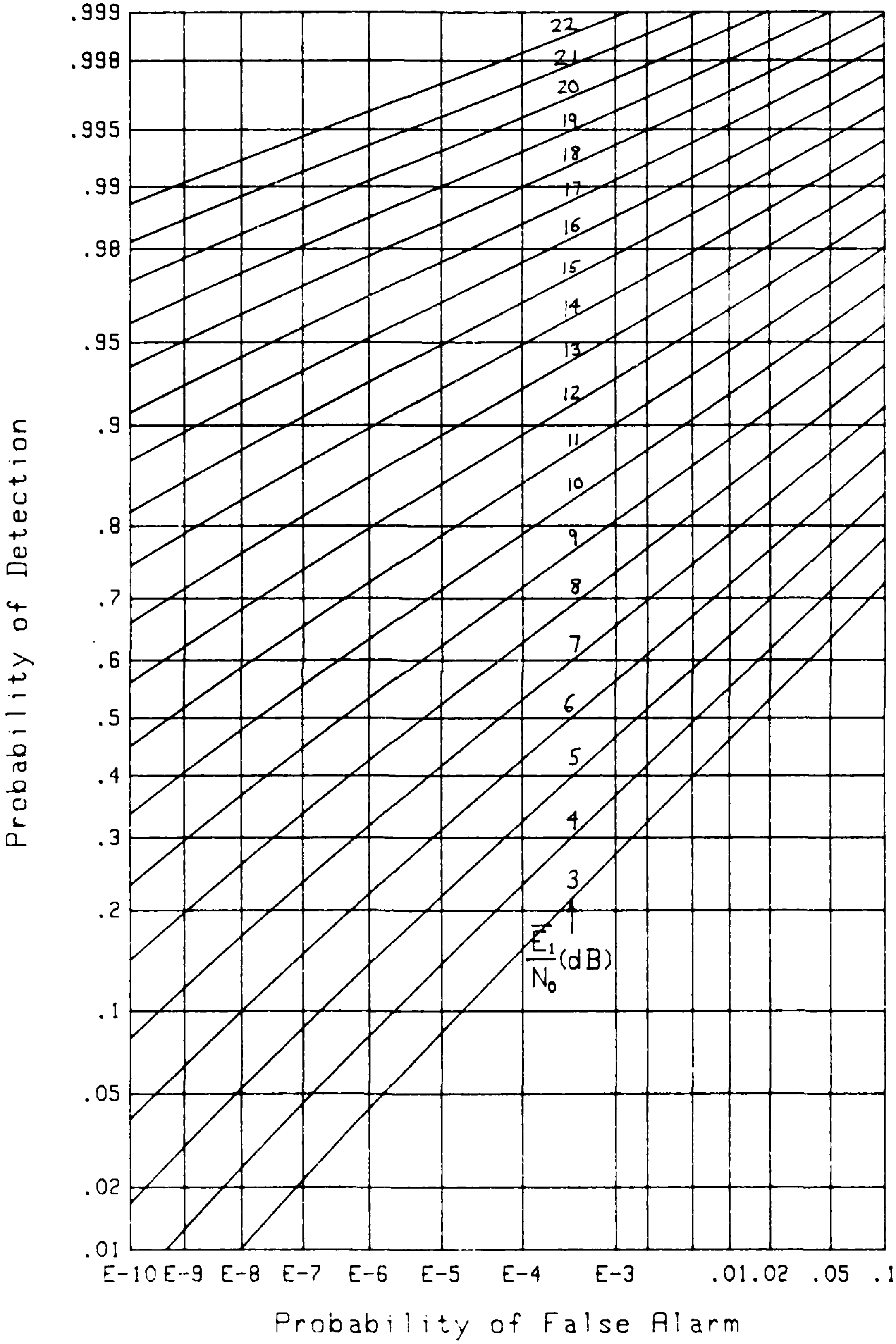

Figure 24. ROC for K=4, m=1, ρ=.5, L=32

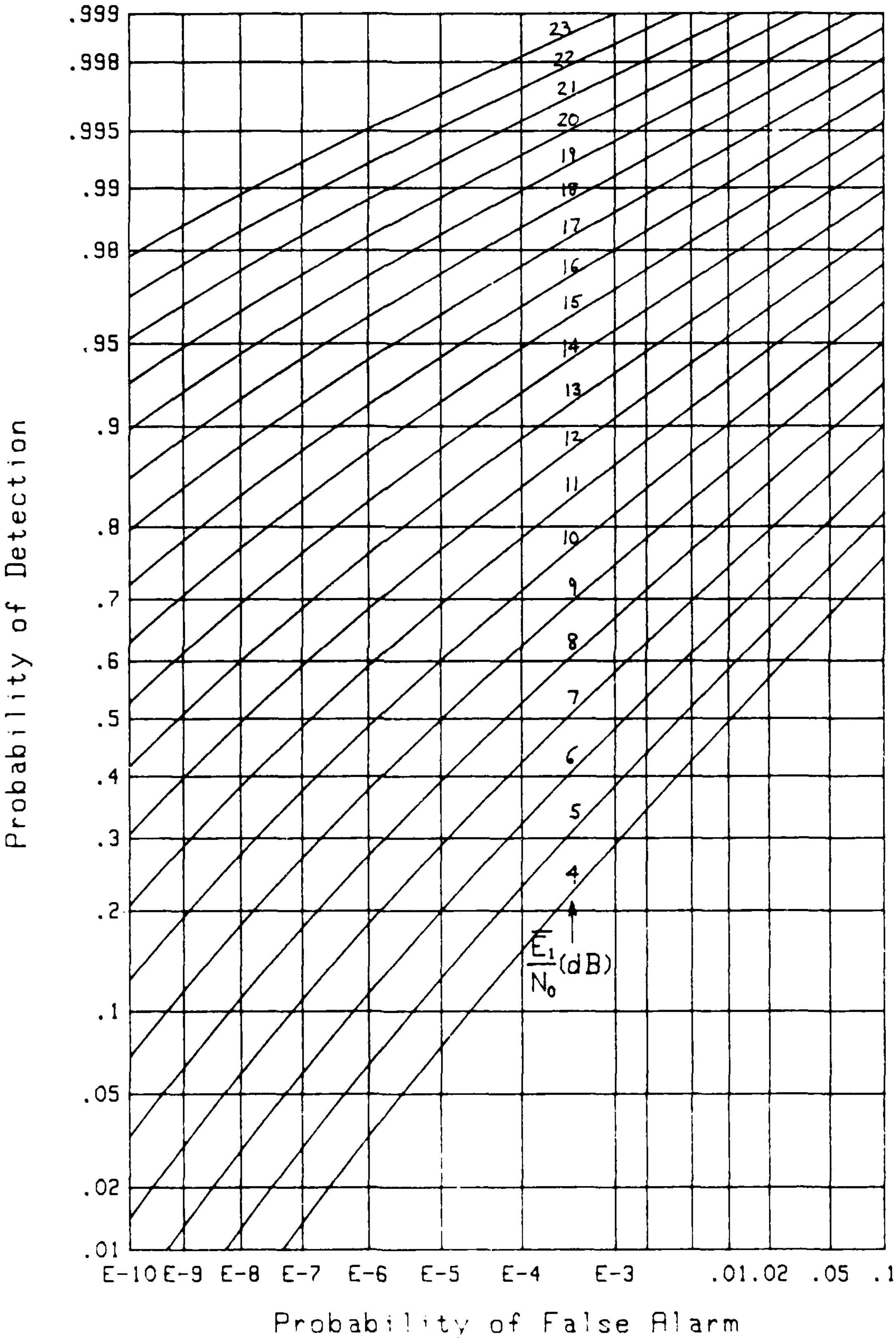

Figure 25. ROC for K=4, m=1, ρ=.5, L=16

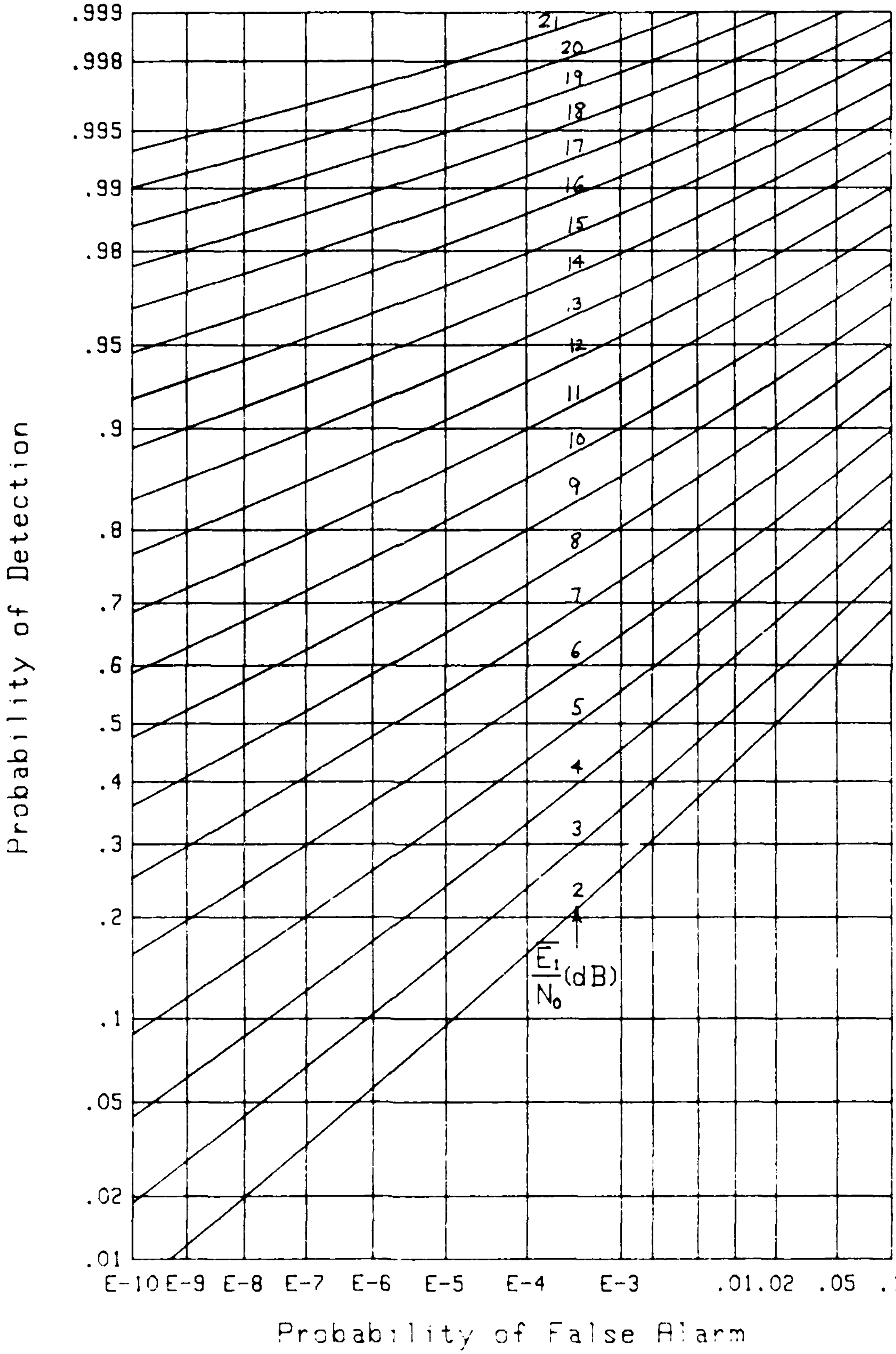

Figure 26. ROC for K=4, m=.5, ρ=0, L=∞

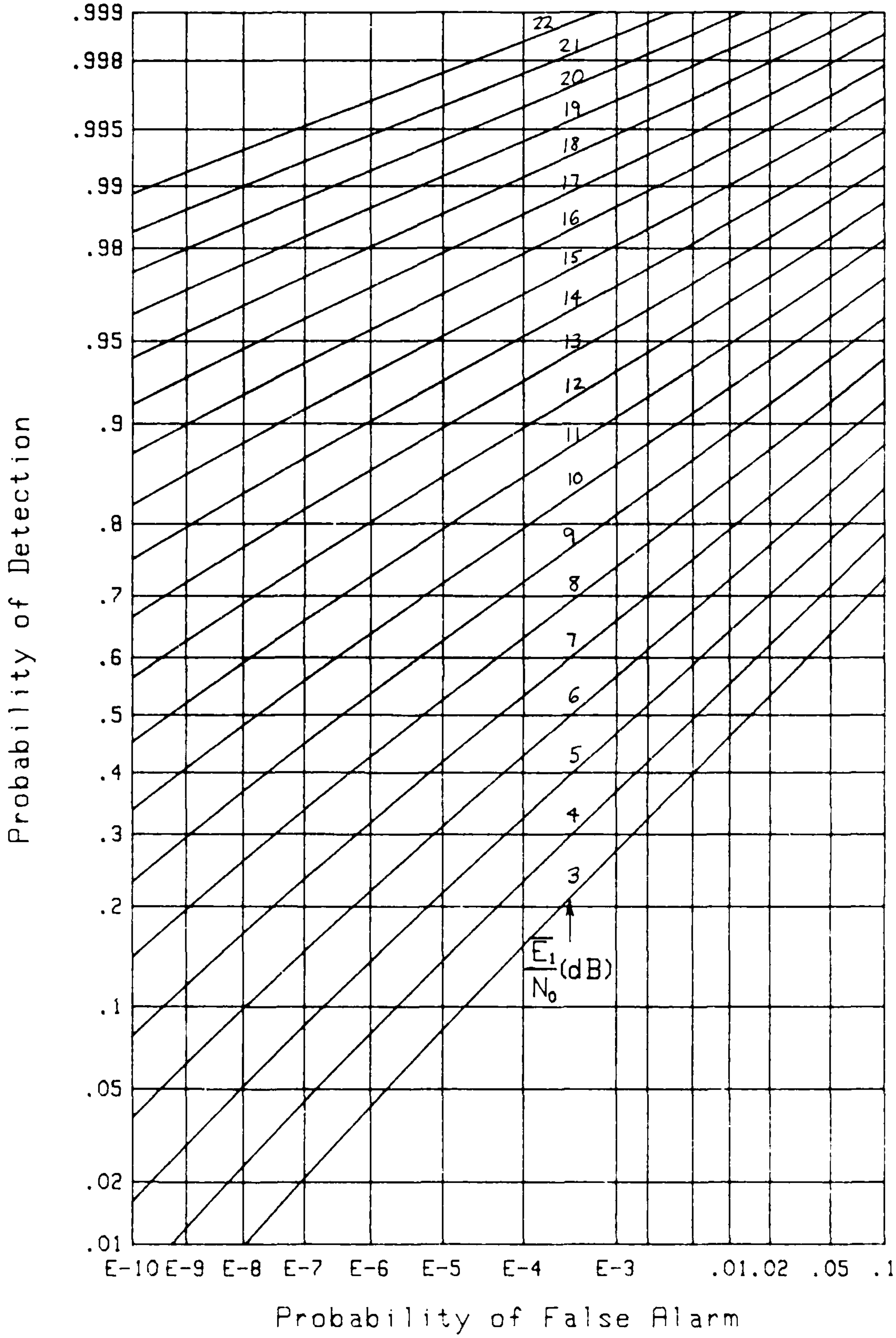

Figure 27. ROC for K=4, m=.5, ρ=0, L=32

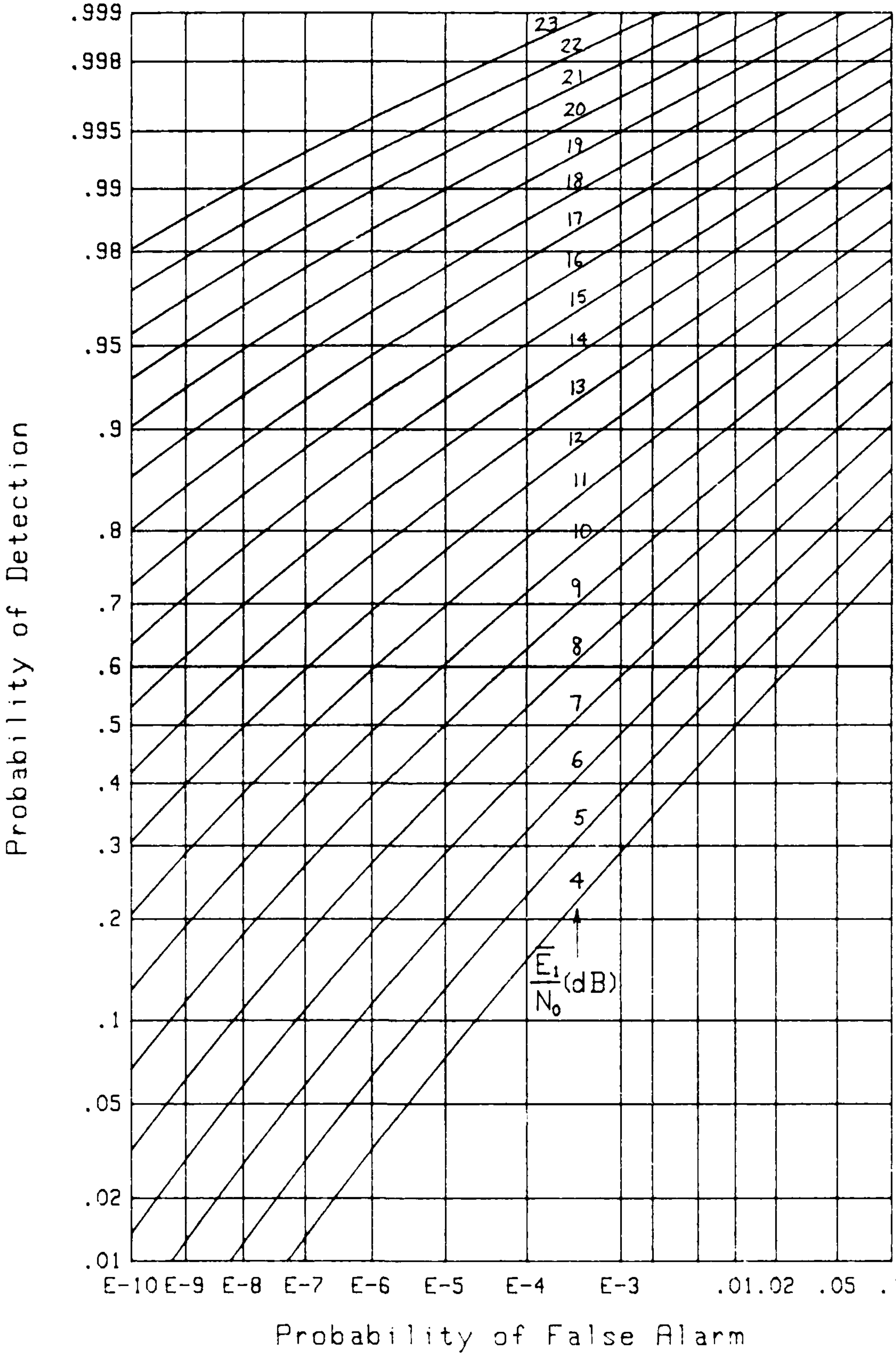

Figure 28. ROC for K=4, m=.5, ρ=0, L=16

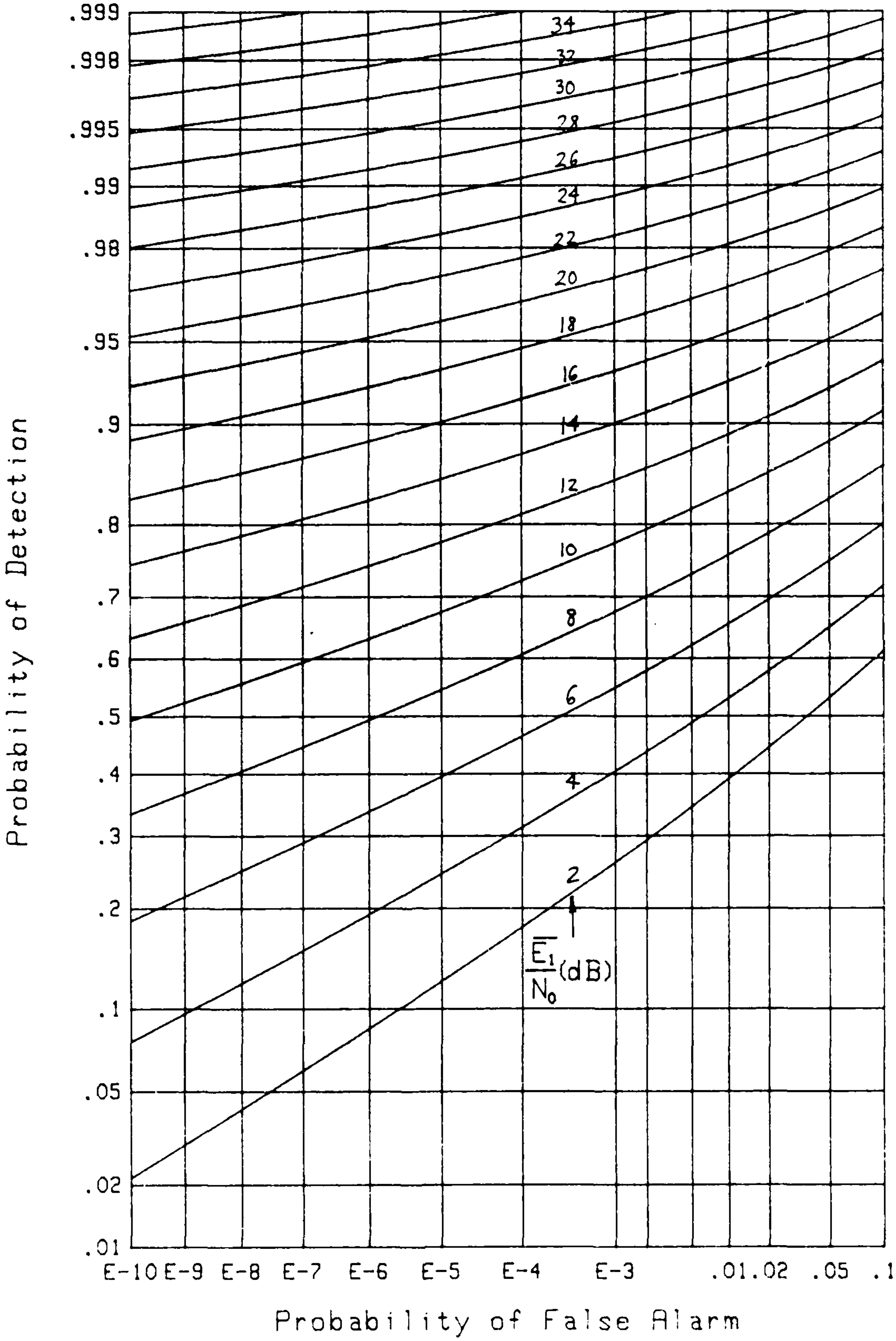

Figure 29. ROC for K=4, m=.5, ρ=.5, L=∞

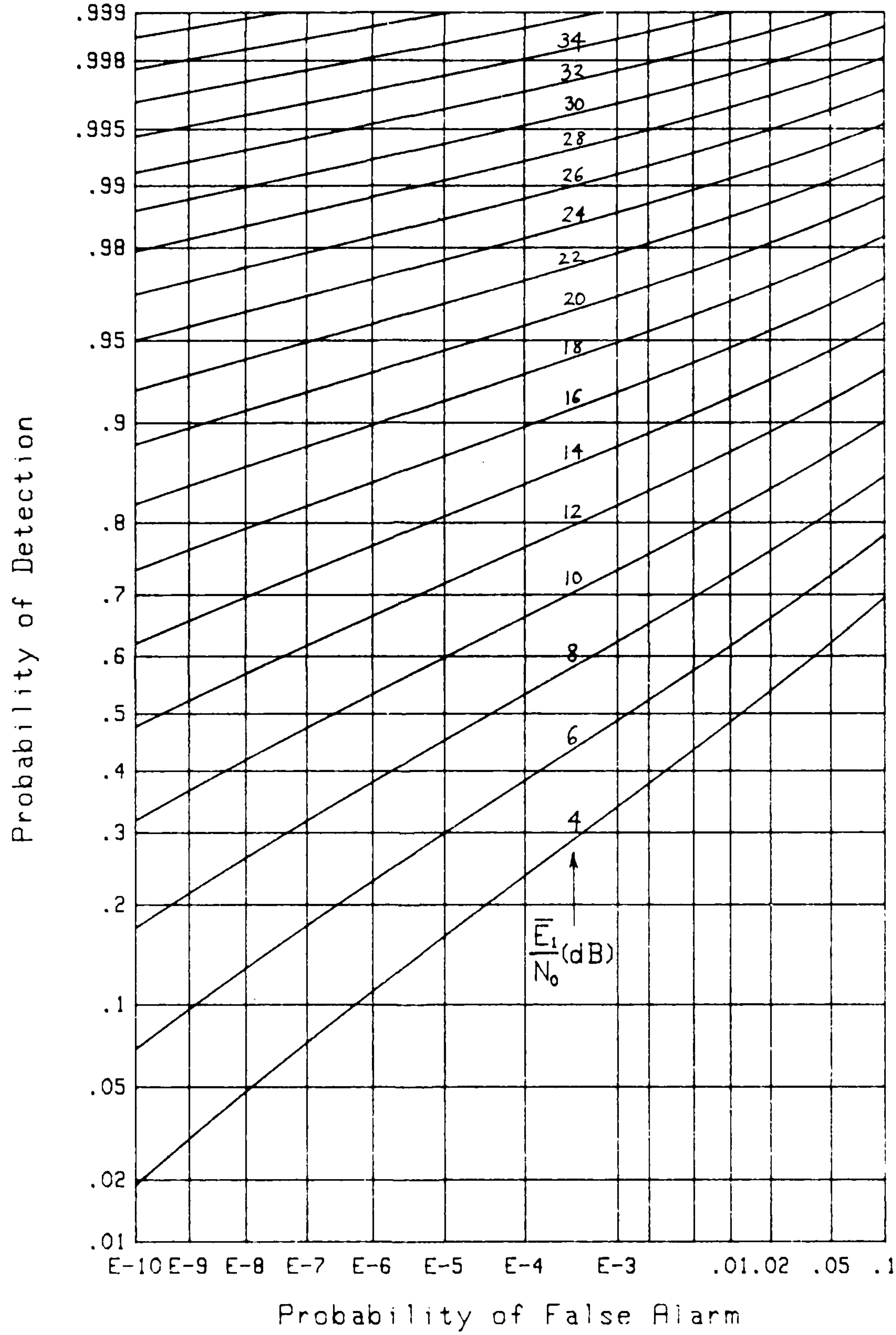

Figure 30. ROC for K=4, m=.5, ρ=.5, L=32

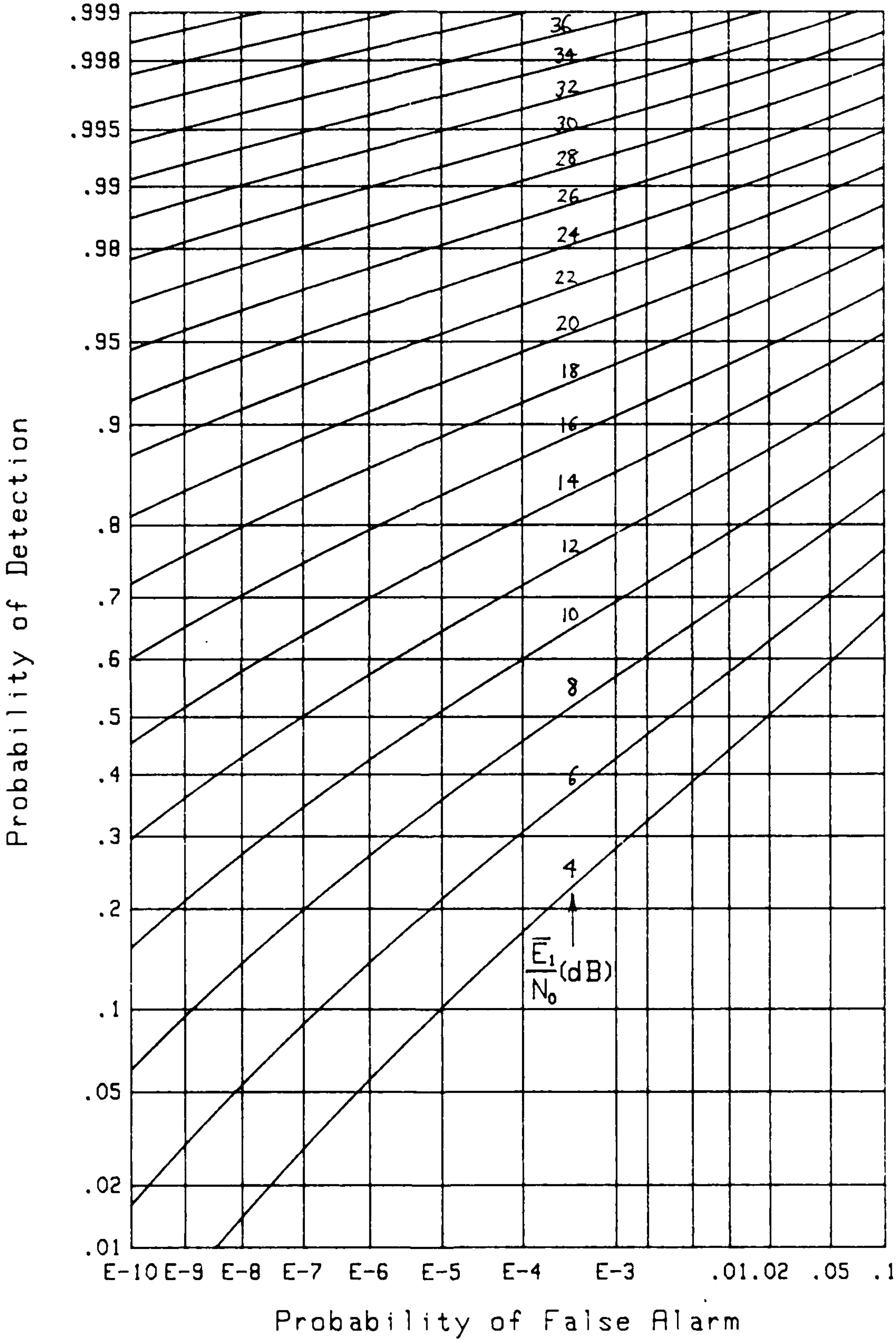

Figure 31. ROC for K=4, m=.5, ρ=.5, L=16

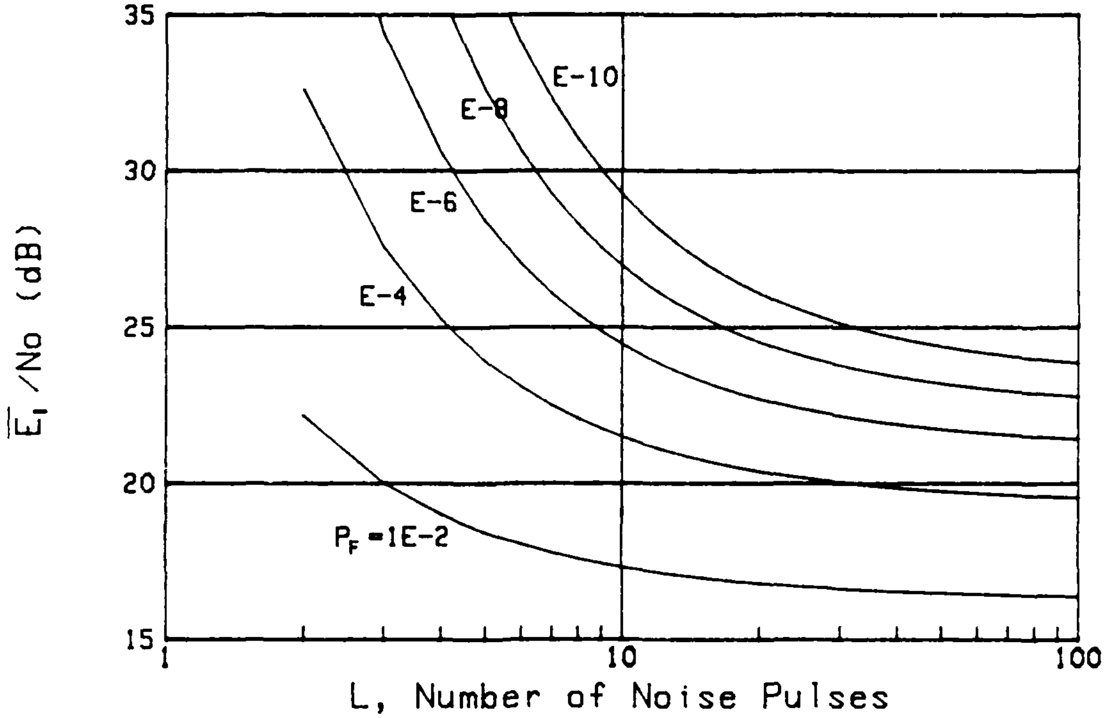

Figure 32. SNR for P_D =.9, K=1, m=1

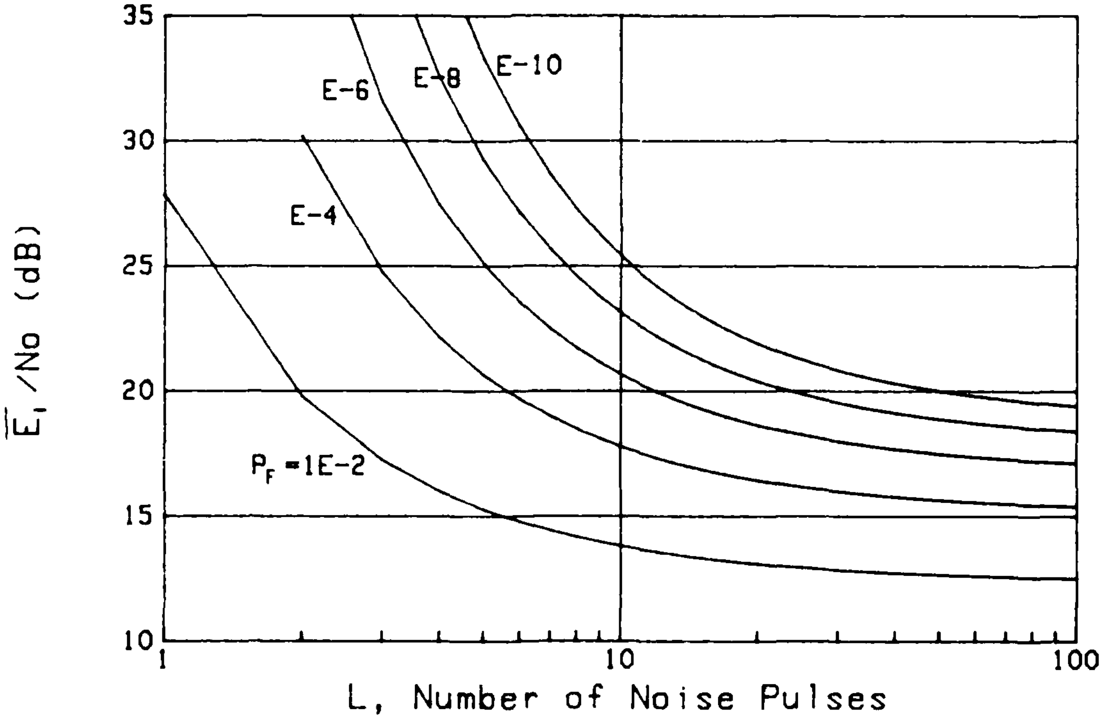

Figure 33. SNR for P_D =.9, K=2, m=1, ρ =.5

APPENDIX A

PROGRAM LISTINGS

There are two programs listed in this appendix, the first for L finite, the second for L infinite, where L is the number of noise-only pulses used to establish a reference. The fundamental parameters K,m,L are input in lines 20, 30, 40, while ρ is input in line 1400. The particular values of $\overline{E_1}/N_0$ (in dB) that are of interest are input in lines 340 and 350. Provision is made for 20 P_D vs P_F curves in lines 60-90; this can easily be changed to accommodate other cases.

The false alarm and detection probabilities are available in lines 1000 and 1130, respectively. The detection probability utilizes R and N as input variables; see table 2. The particular covariance programmed in lines 1390-1430 is exponential, but this, too, can easily be generalized.

To save space, the complete program for L infinite is not listed. Rather, just the essential false alarm and detection probability routines are listed at the end of the appendix; these are obviously not functions of L. The changes required to accommodate this case of infinite L should be obvious.

```
10  ! GENERATE PD-VS-PF NUMBERS FOR FINITE L
20    K=4                          !  NUMBER OF SIGNAL PULSES ADDED, K
30    Ms=.5                        !  FADING PARAMETER, m   (2m DOF)
40    L=16                         !  NUMBER OF NOISE PULSES ADDED  L
50    DIM U(100)
60    COM Pf(100),Pd1(100),Pd2(100),Pd3(100),Pd4(100),Pd5(100)
70    COM Pd6(100),Pd7(100),Pd8(100),Pd9(100),Pd10(100),Pd11(100)
80    COM Pd12(100),Pd13(100),Pd14(100),Pd15(100),Pd16(100),Pd17(100)
90    COM Pd18(100),Pd19(100),Pd20(100)
100   DOUBLE K,L,I,J               !  INTEGERS
110   S=0.
120   FOR I=1 TO K
130   FOR J=1 TO K
140   S=S+FNCov(I,J)               !  NORMALIZED COVARIANCE COEFFICIENTS
150   NEXT J
160   NEXT I
170   Ke=K*K/S                     !  EQUIVALENT NUMBER OF INDEPENDENT FADES
180   N=Ms*Ke                      !  N = m Ke
190   U=0.
200   U=U+.01
210   Pf=FNPf(U,K,L)
220   IF Pf>.1 THEN 200
230   U1=MAX(U-.01,.01)
240   U=U+.01
250   Pf=FNPf(U,K,L)
260   IF Pf>1E-10 THEN 240
270   U2=U
280   Delu=(U2-U1)/100.
290   FOR I=0 TO 100
300   U=U1+Delu*I
310   U(I)=U                       !  THRESHOLD VALUES
320   Pf(I)=FNPf(U,K,L)            !  PROBABILITY OF FALSE ALARM
330   NEXT I
340   FOR J=1 TO 20
350     E1nodb=2*J+2               !  SIGNAL-TO-NOISE RATIO PER PULSE, E1/No (dB)
360   E1no=10.^(.1*E1nodb)
370   R=E1no*K/N
380   FOR I=0 TO 100
390   U=U(I)
400   Pd=FNPd(U,R,N,K,L)           !  PROBABILITY OF DETECTION
410   IF J=1 THEN Pd1(I)=Pd
420   IF J=2 THEN Pd2(I)=Pd
430         !
440         !
590   IF J=19 THEN Pd19(I)=Pd
600   IF J=20 THEN Pd20(I)=Pd
610   NEXT I
620   NEXT J
630   FOR I=0 TO 100
640   Pf(I)=FNInvphi(Pf(I))
650   Pd1(I)=FNInvphi(Pd1(I))
660   Pd2(I)=FNInvphi(Pd2(I))
670     !
680     !
830   Pd19(I)=FNInvphi(Pd19(I))
840   Pd20(I)=FNInvphi(Pd20(I))
850   NEXT I
860   CALL A
870   END
880   !
```

```
890     DEF FNInvphi(X)          !  AMS 55, 26.2.23
900     IF X=.5 THEN RETURN 0.
910     P=MIN(X,1.-X)
920     T=-LOG(P)
930     T=SQR(T+T)
940     P=1.+T*(1.432788+T*(.189269+T*.001308))
950     P=T-(2.515517+T*(.802853+T*.010328))/P
960     IF X<.5 THEN P=-P
970     RETURN P
980     FNEND
990     !
1000    DEF FNPf(U,DOUBLE K,L)  !  FALSE ALARM PROBABILITY
1010    IF U<=0. THEN RETURN 1.
1020    DOUBLE Ls                   !  INTEGER
1030    U1=U+1.
1040    K1=K-1
1050    S=T=EXP(K*LOG(U/U1))
1060    FOR Ls=1 TO L-1
1070    T=T*(K1+Ls)/(Ls*U1)
1080    S=S+T
1090    NEXT Ls
1100    RETURN 1.-S
1110    FNEND
1120    !
1130    DEF FNPd(U,R,N,DOUBLE K,L)  !  DETECTION PROBABILITY
1140    IF U<=0. THEN RETURN 1.
1150    DOUBLE Ls                         !  INTEGER
1160    U1=U+1.
1170    R1=R+1.
1180    U2=U/U1
1190    Ru=R1+U
1200    K2=K-2
1210    Nk=N-K+1
1220    Y=R1/Ru
1230    X=U2*R/R1
1240    X1=X-1.
1250    S=T=EXP(K*LOG(U2)+N*LOG(U1/Ru))
1260    Ho=0.
1270    H=1.
1280    FOR Ls=1 TO L-1
1290    T=T*Y
1300    J=K2+Ls
1310    A=((J+Ls+(Nk-Ls)*X)*H+J*X1*Ho)/Ls
1320    Ho=H
1330    H=A
1340    S=S+T*H
1350    NEXT Ls
1360    RETURN 1.-S
1370    FNEND
1380    !
1390    DEF FNCov(DOUBLE I,J)
1400      Rho=.5                    !  NORMALIZED COVARIANCE COEFFICIENT
1410    Cov=Rho^ABS(I-J)            !  EXPONENTIAL BEHAVIOR
1420    RETURN Cov
1430    FNEND
1440    !
```

```
1450    SUB A    !  PLOT PD VS PF ON NORMAL PROBABILITY PAPER
1460    COM Pf(*),Pd1(*),Pd2(*),Pd3(*),Pd4(*),Pd5(*)
1470    COM Pd6(*),Pd7(*),Pd8(*),Pd9(*),Pd10(*),Pd11(*)
1480    COM Pd12(*),Pd13(*),Pd14(*),Pd15(*),Pd16(*),Pd17(*)
1490    COM Pd18(*),Pd19(*),Pd20(*)
1500    DIM A$[30],B$[30]
1510    DIM Xlabel$(1:30),Ylabel$(1:30)
1520    DIM Xcoord(1:30),Ycoord(1:30)
1530    DIM Xgrid(1:30),Ygrid(1:30)
1540    DOUBLE N,Lx,Ly,Nx,Ny,I      ! INTEGERS
1550    !
1560    A$="Probability of False Alarm"
1570    B$="Probability of Detection"
1580    !
1590    Lx=12
1600    REDIM Xlabel$(1:Lx),Xcoord(1:Lx)
1610    DATA E-10,E-9,E-8,E-7,E-6,E-5,E-4,E-3,.01,.02,.05,.1
1620    READ Xlabel$(*)
1630    DATA 1E-10,1E-9,1E-8,1E-7,1E-6,1E-5,1E-4,.001,.01,.02,.05,.1
1640    READ Xcoord(*)
1650    !
1660    Ly=18
1670    REDIM Ylabel$(1:Ly),Ycoord(1:Ly)
1680    DATA .01,.02,.05,.1,.2,.3,.4,.5,.6,.7,.8,.9
1690    DATA .95,.98,.99,.995,.998,.999
1700    READ Ylabel$(*)
1710    DATA .01,.02,.05,.1,.2,.3,.4,.5,.6,.7,.8,.9
1720    DATA .95,.98,.99,.995,.998,.999
1730    READ Ycoord(*)
1740    !
1750    Nx=14
1760    REDIM Xgrid(1:Nx)
1770    DATA 1E-10,1E-9,1E-8,1E-7,1E-6,1E-5,1E-4
1780    DATA .001,.002,.005,.01,.02,.05,.1
1790    READ Xgrid(*)
1800    !
1810    Ny=18
1820    REDIM Ygrid(1:Ny)
1830    DATA .01,.02,.05,.1,.2,.3,.4,.5,.6,.7,.8,.9
1840    DATA .95,.98,.99,.995,.998,.999
1850    READ Ygrid(*)
1860    !
1870    FOR I=1 TO Lx
1880    Xcoord(I)=FNInvphi(Xcoord(I))
1890    NEXT I
1900    FOR I=1 TO Ly
1910    Ycoord(I)=FNInvphi(Ycoord(I))
1920    NEXT I
1930    FOR I=1 TO Nx
1940    Xgrid(I)=FNInvphi(Xgrid(I))
1950    NEXT I
1960    FOR I=1 TO Ny
1970    Ygrid(I)=FNInvphi(Ygrid(I))
1980    NEXT I
1990    X1=Xgrid(1)
2000    X2=Xgrid(Nx)
2010    Y1=Ygrid(1)
2020    Y2=Ygrid(Ny)
2030    Scale=(Y2-Y1)/(X2-X1)
```

```
2040    GINIT 200. 260.                                 !  VERTICAL PAPER
2050    PLOTTER IS 505,"HPGL"
2060    PRINTER IS 505
2070     PRINT "VS4"
2080    LIMIT PLOTTER 505,0.,200.,0.,260.              !  1 GDU = 2 mm
2090    VIEWPORT 22.,85.,19.,122.
2100  ! VIEWPORT 22.,85.,59.,122.                       !  TOP OF PAPER
2110  ! VIEWPORT 22.,85.,19.,62.                        !  BOTTOM OF PAPER
2120    WINDOW X1,X2,Y1,Y2
2130    FOR I=1 TO Nx
2140    MOVE Xgrid(I),Y1
2150    DRAW Xgrid(I),Y2
2160    NEXT I
2170    FOR I=1 TO Ny
2180    MOVE X1,Ygrid(I)
2190    DRAW X2,Ygrid(I)
2200    NEXT I
2210    CSIZE 2.3,.5
2220    LORG 5
2230    Y=Y1-(Y2-Y1)*.02
2240    FOR I=1 TO Lx
2250    MOVE Xcoord(I),Y
2260    LABEL Xlabel$(I)
2270    NEXT I
2280    CSIZE 3.,.5
2290    MOVE .5*(X1+X2),Y1-.06*(Y2-Y1)
2300    LABEL A$
2310    MOVE .5*(X1+X2),Y1-.1*(Y2-Y1)
2320      LABEL "Figure 31. ROC for K=4, m=.5,  =.5, L=16"
2330    CSIZE 2.3,.5
2340    LORG 8
2350    X=X1-(X2-X1)*.01
2360    FOR I=1 TO Ly
2370    MOVE X,Ycoord(I)
2380    LABEL Ylabel$(I)
2390    NEXT I
2400    LDIR PI/2.
2410    CSIZE 3.,.5
2420    LORG 5
2430    MOVE X1-.15*(X2-X1),.5*(Y1+Y2)
2440    LABEL B$
2450    PENUP
2460    PLOT Pf(*),Pd1(*)
2470    PENUP
2480    PLOT Pf(*),Pd2(*)
2490    PENUP
2500                   !
2510                   !
2820    PLOT Pf(*),Pd19(*)
2830    PENUP
2840    PLOT Pf(*),Pd20(*)
2850    PENUP
2860    PAUSE
2870    PRINTER IS CRT
2880    PLOTTER 505 IS TERMINATED
2890    SUBEND
```

```
10   DEF FNPf(Thr,DOUBLE K)    ! FALSE ALARM PROBABILITY
20   DOUBLE J                   ! INTEGER
30   S=T=EXP(-Thr)
40   FOR J=1 TO K-1
50   T=T*Thr/J
60   S=S+T
70   NEXT J
80   RETURN S
90   FNEND
100  !
110  DEF FNPd(Thr,R,N,DOUBLE K)      ! TR 7707, APP. C-1
120  Error=1.E-10
130  DOUBLE K1,Ks                    ! INTEGERS
140  Et=EXP(Thr)
150  K1=K-1
160  N1=N-1.
170  R1=1.+R
180  Q=R/R1
190  E=Te=1.
200  FOR Ks=1 TO K1
210  Te=Te*Thr/Ks
220  E=E+Te
230  NEXT Ks
240  S=B=MAX(Et-E,0.)
250  T=1.
260  FOR Ks=1 TO 1000
270  Te=Te*Thr/(K1+Ks)
280  B=MAX(B-Te,0.)
290  T=T*Q*(N1+Ks)/Ks
300  Pr=T*B
310  S=S+Pr
320  IF ABS(Pr)<=Error*ABS(S) THEN 350
330  NEXT Ks
340  PRINT "1000 TERMS AT:";K;N;Thr;R;Pr;S
350  Pd=1.-EXP(-Thr-N*LOG(R1))*S
360  RETURN Pd
370  FNEND
```

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